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A Contribution to Understanding Automotive Fuel Economy and Its Limits

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ABSTRACT

The fuel economy of an automobile is a highly complex function of the detailed characteristics of the vehicle and its subsystems (particularly the engine, transmission and drivetrain), as well as being dependent on the manner in which the vehicle is driven. For existing vehicles, automotive manufacturers utilize laboratory test procedures to evaluate fuel economy. However, during new-vehicle design, and to assess the fuel economy potential of new technologies, computer programs that simulate the operation of the vehicle system over prescribed driving schedules are used. Of particular interest are the integrated fuel consumptions on the EPA Urban and Highway driving schedules since these are subject to Federal regulation. Since neither detailed subsystem test data nor simulation programs are typically used by those outside the automotive industry, the physics of fuel economy is not always well understood. This paper presents the physics of motor vehicle fuel economy in an accurate, concise, and understandable form so that meaningful discussion/debate on the prospects for, and the limitations of, fuel economy improvements can be facilitated.

INTRODUCTION

Automotive fuel economy is again a national issue as the regulated Corporate Average Fuel Economy (CAFE) and the levels that must be achieved by automobile manufacturers are being debated. It is driven by a variety of concerns, including the following: 59% of the oil consumed in the U.S. in 2001 was imported (Ref. 1); the possibility of global warming and the contribution that CO₂ produced by motor vehicles might make to it; the environmental impact of exhaust emissions generated by road vehicles.

About 44% of the oil used in the U.S. in 2001 was for motor gasoline (Ref. 1). This is primarily consumed by Light Duty Vehicles (LDVs), and these are the subjects of Federal fuel economy regulation. The objective of this

paper is to present the controlling physics of motor vehicle fuel economy in an accurate, concise and understandable form so that meaningful discussion/debate on the prospects for, and limitations of, fuel economy improvements can be facilitated. Specific recommendations for improving fuel economy will not be made. Furthermore, collateral effects on exhaust emissions or cost will not be addressed.

In this paper, the Federal mandates for vehicle fuel economy are briefly reviewed. A methodology is then described for decomposing a driving schedule into three driving modes, leading to the development of a comprehensive fuel-consumption equation. This equation contains seven key parameters. Means by which these parameters could be changed to decrease vehicle fuel consumption are discussed. The limits of the parameters are also noted. The methodology is then utilized to assess the one used in a recent fuel economy study by the National Research Council. Finally, the fuel-consumption equation is extended to accommodate regenerative braking, leading to a consideration of hybrid powertrains.

DEFINITIONS AND FEDERAL MANDATES

The fuel consumption of any vehicle is strongly dependent on the manner in which it is driven. A driving pattern or schedule is defined by the variation of vehicle velocity, V , with time, t , during a trip. The Environmental Protection Agency (EPA) has selected two particular schedules (Ref. 2) as the basis for fuel economy regulation – Urban and Highway (see Figure 1).

The Urban schedule has 18 starts and stops, a maximum driving speed of 91.2 km/h (56.7 mph), and an average speed while a vehicle is in motion of 38.3 km/h (23.8 mph).

The Highway schedule has only one start and one stop, a maximum driving speed of 96.4 km/h (59.9 mph), and an average speed while a vehicle is in motion of 78.1 km/h (48.5 mph).

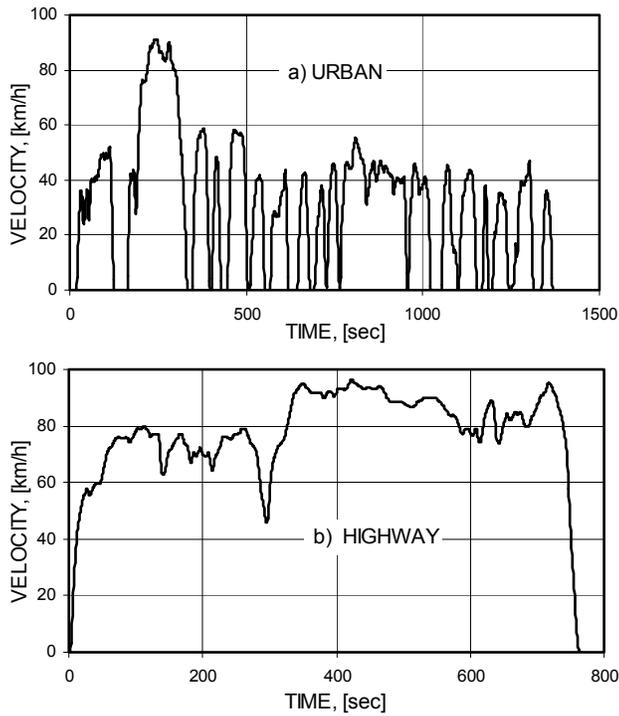


Figure 1. The EPA Driving Schedules

In the transient driving represented by these schedules the instantaneous fuel rate varies with time. The *average* value of the volume of fuel consumed per unit distance traveled is^{1,2}

$$\text{Fuel Consumption} \equiv \tilde{g} = \frac{G_f [\text{gallons}]}{S [\text{miles}]} \quad (1)$$

where G_f is the total volume of fuel consumed in a trip of length S . Its reciprocal is

$$\text{Fuel Economy} \equiv \tilde{m} = \frac{S [\text{miles}]}{G_f [\text{gallons}]} \quad (2)$$

Using subscripts 0 and 1 to designate a vehicle's baseline and improved configurations, respectively, a fuel economy increase ($\% \tilde{m}$) is related to its corresponding fuel consumption reduction ($\% \tilde{g}$) by

$$(\% \tilde{m}) \equiv 100 \left[\frac{\tilde{m}_1 - \tilde{m}_0}{\tilde{m}_0} \right] = \frac{100 \left[\frac{\tilde{g}_0 - \tilde{g}_1}{\tilde{g}_0} \right]}{1 - \left[\frac{\tilde{g}_0 - \tilde{g}_1}{\tilde{g}_0} \right]} = \frac{(\% \tilde{g})}{1 - \frac{(\% \tilde{g})}{100}} \quad (3)$$

¹ The \equiv sign indicates an identity, or an expression defining some quantity.

² A complete nomenclature is given at the end of the paper.

Any percentage reduction in \tilde{g} translates to a greater percentage increase in \tilde{m} . For example, a 33.3% reduction in \tilde{g} translates to a 50% increase in \tilde{m} .

The quantity subject to regulation for any vehicle is the particular weighted combination of its Urban (\tilde{g}_U) and Highway (\tilde{g}_H) fuel consumptions called *combined* fuel consumption,

$$g_c \equiv 0.55 \tilde{g}_U + 0.45 \tilde{g}_H \quad (4)$$

Large automobile companies produce more than one line of Light Duty Vehicles. The EPA divides these vehicles into two generic fleets – passenger cars, and light trucks (small pickups, minivans and SUVs). The CAFE of each fleet is the *reciprocal* of its sales-weighted, *combined* fuel consumption

$$g_{\text{fleet}} \equiv \sum_{i=1}^n (g_c)_i x_i \quad (5)$$

where $(g_c)_i$ is the combined fuel consumption of vehicle line i in the fleet, x_i the fraction of total fleet sales that it comprises, and n the number of vehicle lines in the manufacturer's fleet.

The current CAFE mandates are:

$$\text{passenger cars: } m_{\text{fleet}} \equiv \frac{1}{g_{\text{fleet}}} = 27.5 \frac{\text{miles}}{\text{gallon}} \quad (6)$$

$$\text{light trucks: } m_{\text{fleet}} \equiv \frac{1}{g_{\text{fleet}}} = 20.7 \frac{\text{miles}}{\text{gallon}}$$

The sales of the light-truck category have been progressively increasing, and currently represent about 50% of the market.

Through system design, mainline automobile manufacturers control the $(g_c)_i$ of each vehicle line, but the x_i is determined by the consumer, whose vehicle choice is affected by many factors, not all of them technical. Therefore, the following analysis will be confined to consideration of the factors affecting $(g_c)_i$. Also, the CAFE standards *per se* will not be addressed.

FUEL-CONSUMPTION MODEL

The physics of fuel economy is best analyzed in terms of fuel consumption, and fuel consumption essentially resolves to a matter of demand and supply. Looking first at the demand side and referring to Figure 1, vehicle velocities are specified versus time in the EPA driving schedules, along with the total duration, T , and the idle time, t_{idle} . The propulsive power a vehicle needs to drive these schedules depends on the tractive force required

at the tire/road interface of the driving tires in order to generate the prescribed variations in vehicle speed.

Tractive force relates to the conceptualization of a vehicle in which the powertrain is decoupled from the driving wheels, and all 4 wheels are free and supported by frictionless bearings (see Figure 2). It is the instantaneous "push" or "pull" required to propel that vehicle at any prescribed velocity and acceleration. It must overcome the retarding forces generated by rolling resistance of the tires and aerodynamic drag of the body, as well as produce any required vehicle acceleration. It is the force that has to be provided on the supply side of a vehicle system by the powertrain.

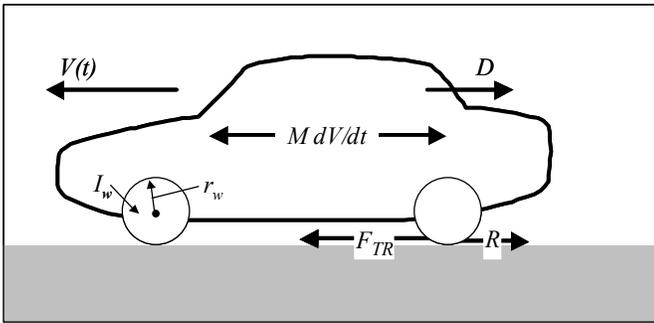


Figure 2. Forces acting on a vehicle driving at varying speed on a smooth, level road in the absence of ambient wind.

Instantaneously, the required tractive force F_{TR} for driving on a smooth, level road in the absence of ambient wind is

$$F_{TR} = R + D + \left(M + 4 \left[\frac{I_w}{r_w^2} \right] \right) \left(\frac{dV}{dt} \right) \quad (7)$$

$$= \underbrace{r_0 M g}_{\text{tire resistance}} + \underbrace{C_D A \frac{V^2}{2} \rho}_{\text{aerodynamic drag}} + \underbrace{\left(M + 4 \left[\frac{I_w}{r_w^2} \right] \right) \left(\frac{dV}{dt} \right)}_{\text{linear + rotational inertia}}$$

where r_0 is the tire rolling resistance coefficient, C_D the aerodynamic drag coefficient, A the vehicle frontal area, M the vehicle mass, ρ the density of ambient air, and $g = 9.81 \text{ m/s}^2$ is the gravitational constant. The r_0 and C_D coefficients are assumed to be independent of vehicle speed. The term $4 \left(\frac{I_w}{r_w^2} \right)$ accounts for the fact that the 4 rotating wheels must be angularly as well as linearly accelerated, where I_w is the polar moment of inertia of a wheel assembly, and r_w its effective rolling radius³.

³ The sum $(R+D)$ is the so-called road-load force. The Recommended Practice of SAE J1263 defines a

The tire resistance, R , and the aerodynamic drag, D , are never negative, but the inertia force can be negative. Consequently, there are three possibilities:

$F_{TR} > 0$: This occurs when $dV/dt > 0$ (vehicle acceleration), or when $dV/dt = 0$ (constant-speed driving). It can also occur when $dV/dt < 0$ (vehicle deceleration) as long as the magnitude of the inertia force does not exceed $(R+D)$. This represents a powered deceleration, a term that might seem like an oxymoron.

$F_{TR} < 0$: This occurs when the magnitude of the inertia force in a deceleration is required to be of greater magnitude than $(R+D)$, i.e., when a retarding or braking force is required at the tire/road interface of the wheels.

$F_{TR} = 0$: No tractive force is required when a vehicle is stationary. When a vehicle is moving, $F_{TR} = 0$ occurs if the magnitude of the inertia force is equal to $(R+D)$. Such coasting⁴ requires a segment of $V(t)$ in which the velocity profile uniquely matches the r_0 , C_D , A , M , and wheel-assembly characteristics of a vehicle, and so a $V(t)$ that prescribes $F_{TR} = 0$ for one vehicle will not do so for another. This suggests that coasting, or even a close approximation to it, is unlikely on the EPA schedules.

This breakdown suggests that the EPA schedules be decomposed into these three generic modes for fuel-consumption analysis. The objective will be to formulate an integral equation for the total fuel consumed that can be reduced to algebraic form by introducing physically-based averages for some of the parameters.

coastdown procedure for measuring "road-load force" for vehicles. However, while the transmission of the vehicle is in neutral during the coastdown, the drivetrain is still connected to the driving wheels and its reaction to the vehicle deceleration is a retarding force on the vehicle. This "braking" is in addition to $(R+D)$, and so would have to be properly accounted for if the $(R+D)$ is to be determined from the coastdown.

⁴ This is to be distinguished from the "coasting" that occurs when a driver fully releases the gas pedal in a moving vehicle in order to accomplish a deceleration. In this case, vehicle inertia tends to drive the powertrain and the reaction to this is a braking force ($F_{TR} < 0$) on the vehicle. Alternatively, engine power at closed throttle may still be sufficient to deliver a small tractive force ($F_{TR} > 0$) to the vehicle.

FUEL-CONSUMPTION EQUATION

The fuel consumption for a driving schedule is:

$$\begin{aligned} \tilde{g} &\equiv \frac{G_f}{S} = \frac{1}{S} \left(\frac{m_f}{\rho_f} \right) \\ &= \frac{1}{S\rho_f} \left[\underbrace{(m_f)^+}_{F_{TR}>0} + \underbrace{(m_f)_{brkg}}_{F_{TR}<0} + \underbrace{(m_f)_{idle}}_{F_{TR}=0} \right] \end{aligned} \quad (8)$$

where m_f represents a mass of fuel consumed, and ρ_f the fuel's density. The symbol + is used throughout this paper to denote the powered driving mode. The fuel-mass components in this equation will be examined one at a time.

POWERED DRIVING ($F_{TR} > 0$)

Since vehicle speed varies with time during this driving mode, both the instantaneous engine power required to produce vehicle propulsion and the corresponding rate of fuel consumption also vary. In order to determine the total mass of fuel consumed, the fuel rate has to be integrated over the duration of the mode.

$$(m_f)^+ \equiv \int \dot{m}_f dt \quad (9)$$

where \dot{m}_f is the instantaneous mass fuel rate, and + represents the duration of powered driving (τ) in a schedule. Since τ is not known *a priori*, it needs to be determined.

The instantaneous brake thermal efficiency of an engine is

$$\eta_b \equiv \frac{\text{power out}}{\text{energy rate in}} = \frac{P_b}{\dot{m}_f H_f} \quad (10)$$

where P_b is the brake power at the output shaft and H_f the heating value of the fuel per unit mass.

The total work or brake energy delivered by the engine during all of the powered driving segments is,

$$(E_b)^+ = \int dE_b = \int P_b dt \quad (11)$$

where dE_b is an increment of brake energy (work) delivered in a time increment dt ,

Substituting the P_b from Equation (10) gives,

$$(E_b)^+ = H_f \int \eta_b \dot{m}_f dt = H_f \int \eta_b dm_f \quad (12)$$

where dm_f is the incremental mass of fuel, $\dot{m}_f dt$, consumed to produce dE_b .

The engine operating point varies over broad ranges of torque and speed during this driving mode and so η_b also varies. To evaluate the integral, a mean value of η_b is defined such that

$$(E_b)^+ = H_f \tilde{\eta}_b \int dm_f = H_f \tilde{\eta}_b (m_f)^+ \quad (13)$$

Solving for the total mass of fuel consumed during powered driving,

$$(m_f)^+ = \frac{(E_b)^+}{H_f \tilde{\eta}_b} \quad (14)$$

The specific nature of $\tilde{\eta}_b$ is revealed by solving Equation (13) for $\tilde{\eta}_b$ and substituting the expression for $(E_b)^+$ from Equation (12). This gives,

$$\tilde{\eta}_b \equiv \frac{(E_b)^+}{H_f (m_f)^+} \equiv \frac{1}{(m_f)^+} \int \eta_b dm_f \quad (15)$$

Hence, $\tilde{\eta}_b$ is a fuel-consumption-weighted average, not a time average. Consequently, the efficiency at the engine operating points of higher fuel consumption during powered driving have the greatest influence on its value. The importance of this will become apparent later.

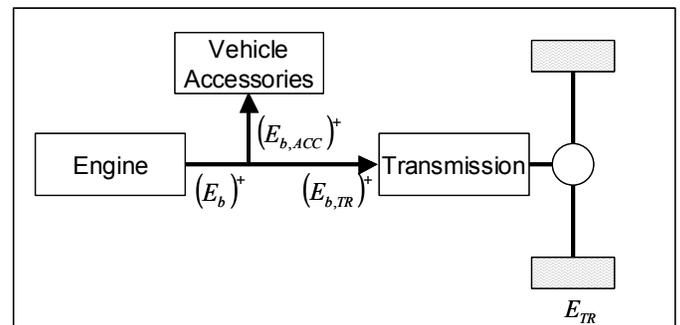


Figure 3. Energy-Flow Diagram for the Duration of Powered Driving

Equation (14) requires an expression for $(E_b)^+$. The energy-flow diagram for a powertrain is shown in Figure 3 where, for clarity, a rear-wheel drive configuration is shown even though most cars today have front-wheel drive. At any instant the tractive energy required at the tire-road interfaces of the driving wheels to provide vehicle propulsion is $F_{TR}Vdt$. Its integral over the duration of powered driving is the total required tractive

energy E_{TR} . This requires that $(E_{b,TR})^+$ be delivered to the drivetrain.

In a manner similar to that used in defining $\tilde{\eta}_b$ in Equation (15), an average drivetrain efficiency can be defined as

$$\tilde{\eta}_{dr} \equiv \frac{E_{TR}}{(E_{b,TR})^+} \equiv \frac{1}{(E_{b,TR})^+} \int \eta_{dr} dE_{b,TR} \quad (16)$$

In this case, it is an energy-transfer-weighted average of the instantaneous drivetrain efficiency, η_{dr} .

Energy, $(E_{b,ACC})^+$, is required by vehicle accessories. These represent subsystems that require energy but are not necessary for propulsion. Some of them function during fuel-economy evaluation (e.g., electrical loads for instrument-panel display, computer control systems, daytime-running and brake lights) while others are inoperative but impose tare loads on the engine (e.g., power steering, de-clutched A/C compressor). Their total energy requirement during powered driving is $(E_{b,ACC})^+$.

The total brake energy required from the engine is therefore

$$(E_b)^+ = (E_{b,TR})^+ + (E_{b,ACC})^+ \quad (17)$$

Using Equation (16) to replace $(E_{b,TR})^+$ in Equation (17), and putting the result into Equation (14) gives,

$$(m_f)^+ = \frac{1}{\underbrace{H_f \eta_{b,max} \left(\frac{\tilde{\eta}_b}{\eta_{b,max}} \right)}_{\text{Brake-Energy Supply (per unit mass of fuel)}} \underbrace{\left[\frac{E_{TR}}{\tilde{\eta}_{dr}} + (E_{b,ACC})^+ \right]}_{\text{Brake-Energy Demand}}} \quad (18)$$

where $\eta_{b,max}$, the maximum brake thermal efficiency in the operating range of an engine, is introduced. This gives two engine-efficiency parameters⁵. The virtue of this will become evident later.

BRAKING ($F_{TR} < 0$)

The fuel-mass component for braking is obtained from

$$(m_f)_{brkg} = \int_{brkg} (\dot{m}_f)_{brkg} dt \quad (19)$$

⁵ If brake specific fuel consumption (b) is preferred over η_b , the following alternatives can be used in Equation (18):

$\eta_{b,max} \equiv 1/(b_{min} H_f)$ and $\tilde{\eta}_b/\eta_{b,max} \equiv b_{min}/\tilde{b}$. In this case, \tilde{b} is a brake-energy-weighted average.

The fuel rate generally varies during braking, so its time-averaged value will be used.

$$(m_f)_{brkg} = (\overline{\dot{m}_f})_{brkg} t_{brkg} \quad (20)$$

where,

$$t_{brkg} = T - \tau - t_{idle} \quad (21)$$

Again, T is the specified total duration of a driving schedule, and t_{idle} its specified idle time. As already stated, τ is the duration of powered driving, and has to be determined.

IDLING ($F_{TR} = 0$)

The fuel-mass component for idling is

$$(m_f)_{idle} = \int_{idle} (\dot{m}_f)_{idle} dt \quad (22)$$

The fuel rate is essentially constant during idling. Therefore,

$$(m_f)_{idle} = (\dot{m}_f)_{idle} t_{idle} \quad (23)$$

DRIVING-SCHEDULE FUEL CONSUMPTION

Putting these fuel consumptions for the three driving modes (Equations (18), (20), and (23)) into in Equation (8) provides the following expression for driving-schedule fuel consumption.

$$\tilde{g} = \frac{1}{S \rho_f} \left\{ \frac{\left[\frac{E_{TR}}{\tilde{\eta}_{dr}} + (E_{b,ACC})^+ \right]}{H_f \eta_{b,max} \left(\frac{\tilde{\eta}_b}{\eta_{b,max}} \right)} + (\dot{m}_f)_{brkg} (T - \tau - t_{idle}) + (\dot{m}_f)_{idle} t_{idle} \right\} \quad (24)$$

For \tilde{g} in liters/km, the following units⁶ are required: S [km], ρ_f [kg/liter], H_f [MJ/kg], E [MJ], \dot{m}_f [kg/s], t [s].

This equation is physics-based, and accounts for *all* of the fuel consumed during a driving schedule. It is the basis on which the following analyses of fuel-consumption reduction will be made. Fuel consumption

⁶ Metric units, the standard of the U.S. automotive industry, are indicated here; however, everywhere else in the paper the fuel consumption values are given in gal/mile, the units corresponding to those of the regulated U.S. fuel economy levels.

cannot change unless at least one of the parameters in Equation (24) changes.

Five of the parameters in this fuel-consumption equation are vehicle independent, and have fixed values.

S : The total distance traveled in the EPA driving schedules is (Ref. 2):

$$S_U = 12.00 \text{ km}$$

$$S_H = 16.50 \text{ km}$$

ρ_f : The density of gasoline is nearly independent of octane rating and has a value of about 0.735 kg/liter (corresponding to a specific gravity of 0.735). The specific gravity of diesel fuel is about 0.845, which is 15% greater. This gives diesel-powered vehicles a 13% advantage in volumetric fuel consumption, and a full 15% advantage in volumetric fuel economy (Equation (3)).

H_f : The heating value per unit mass of gasoline is nearly independent of octane rating, with a value of about 42.7 [MJ/kg]. The heating value of diesel fuel is very nearly the same.

T : The total duration of the EPA schedules is (Ref. 2):

$$T_U = 1369 \text{ s}$$

$$T_H = 765 \text{ s}$$

t_{idle} : The prescribed idle time in the EPA schedules is (Ref. 2):

$$(t_{idle})_U = 241 \text{ s}$$

$$(t_{idle})_H = 4 \text{ s}$$

FUEL-CONSUMPTION ANALYSIS

The component fuel consumptions in Equation (24) will be examined one at a time.

POWERED DRIVING

This component involves the brake-energy demand of the vehicle (the numerator) and the brake-energy supply of the engine (the denominator). The demand side will be considered first. Its dominant term is the total tractive energy, E_{TR} , for this driving mode on the EPA schedules. The tractive-energy concept was published in 1981 (Ref. 3), and was later re-evaluated and extended by the present authors.

Tractive Energy, E_{TR}

The instantaneous tractive force needed to move a vehicle is given by Equation (7); hence, the corresponding tractive power is

$$\begin{aligned} P_{TR} &= F_{TR} V \\ &= \left[r_0 M g + C_D A \frac{V^2}{2} \rho + \left(M + 4 \left[\frac{I_w}{r_w^2} \right] \right) \left(\frac{dV}{dt} \right) \right] V \\ &= f \left\{ \underbrace{r_0, C_D, A, M, I_w, r_w}_{\text{vehicle}}, \underbrace{V, (dV/dt)}_{\text{driving schedule}} \right\} \end{aligned} \quad (25)$$

A typical variation of P_{TR} is shown in Figure 4 during one of the start/stop segments of the EPA Urban Schedule. In the power plot, the areas between the curve and the zero axis are energies. Those above the axis are tractive energy that has to be supplied by the powertrain. Those below the axis are energy that must be removed from the vehicle by wheel and powertrain braking.

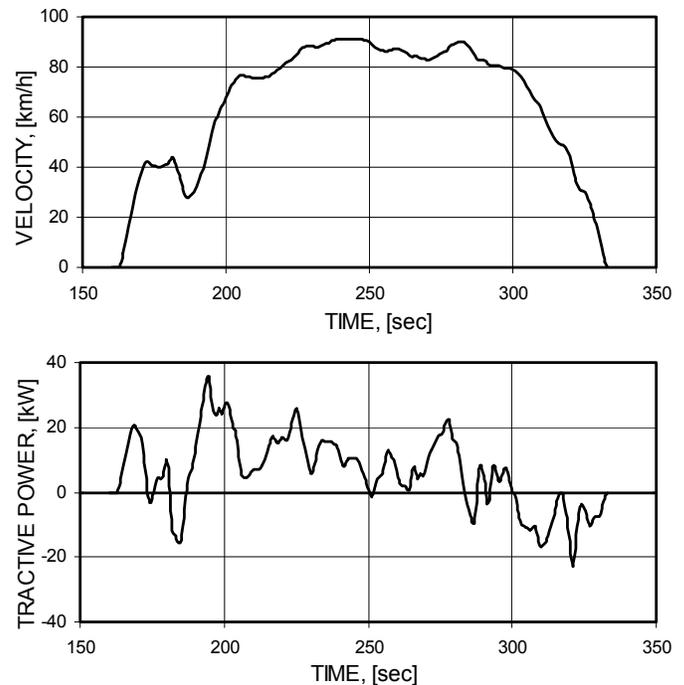


Figure 4. Velocity and a Typical Variation of Tractive Power during a Start/Stop Segment of the EPA Urban Schedule.

A typical plot of tractive power for the complete Urban Schedule is shown in Figure 5. This driving schedule requires a large number of tractive and braking segments.

Summing up the energies of all the tractive segments in a complete schedule, and doing the same for the braking segments, the two sums are very well represented by the following linear equations and the coefficients in Table 1:

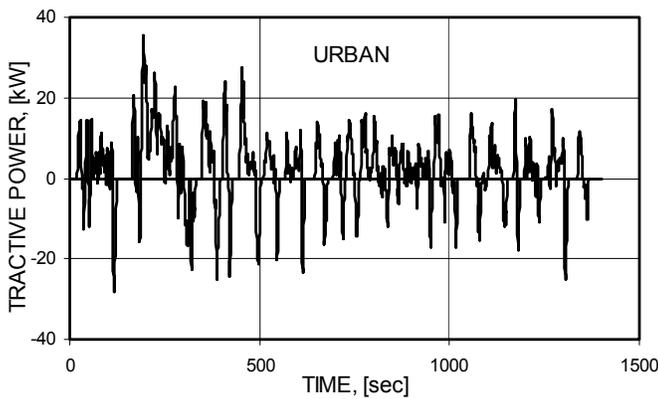


Figure 5. Typical Tractive-Power Requirement for the EPA Urban Schedule

$$\frac{E_{TR}}{MS} = \underbrace{\alpha r_0}_{\text{energy for overcoming rolling resistance}} + \underbrace{\beta \left(\frac{C_D A}{M} \right)}_{\text{energy for overcoming aerodynamic drag}} + \underbrace{\gamma \left(1 + \frac{4 I_w}{M r_w^2} \right)}_{\text{energy to produce increases in vehicle kinetic energy}} \quad (26)$$

$$\frac{E_{BR}}{MS} = \underbrace{\gamma' \left(1 + \frac{4 I_w}{M r_w^2} \right)}_{\text{required decrease in vehicle kinetic energy}} - \underbrace{\alpha' r_0}_{\text{energy dissipated by tires}} - \underbrace{\beta' \left(\frac{C_D A}{M} \right)}_{\text{energy dissipated by aerodynamic drag}} \quad (27)$$

a) TRACTIVE ENERGY		
Coefficient	Urban	Highway
α [m/s^2]	7.439	9.074
β [kg/ms^2]	110.0	293.6
γ [m/s^2]	0.1515	0.0403
FIT: r^2	0.99991	0.99997

b) BRAKING ENERGY		
Coefficient	Urban	Highway
α' [m/s^2]	2.371	0.7360
β' [kg/ms^2]	21.52	16.79
γ' [m/s^2]	0.1515	0.0403
FIT: r^2	0.9985	0.9921

$$M = [\text{kg}]; \quad A = [\text{m}^2]; \quad E_{TR}, E_{BR} = [\text{kJ}]; \quad S = [\text{km}]; \\ I_w = [\text{kg m}^2]; \quad r_w = [\text{m}]$$

Table 1. Coefficients for Tractive and Braking Energy on the EPA Urban and Highway Schedules.

The coefficients were statistically developed based on a wide range of vehicle parameters and an air density⁷ of

⁷ For ρ different than 1.20 [kg/m^3], β and β' must be corrected by the multiplier ($\rho/1.20$).

1.20 [kg/m^3]. They are slightly different than those in Reference 3. These E_{TR} and E_{BR} correlations and their coefficients permit the impact of changes in r_0 , C_D , A , M , or wheel-assembly rotating inertia on fuel consumption to be evaluated.

For a representative Midsize Car⁸ the tractive energy on the two driving schedules is split as indicated in Table 2.

	Tires	Aero	Inertia
Urban	25%	17%	58%
Highway	33%	50%	17%

Table 2. Tractive-Energy Splits on the EPA Driving Schedules [$r_0=0.009$, $C_D=0.338$, $A=2.06 \text{ m}^2$, $M=1644 \text{ kg}$, $I_w=0.949 \text{ kg m}^2$, $r_w=0.320 \text{ m}$]

On the Urban schedule, the dominant term is inertia because of the many accelerations. The smallest term is aero because of the low average driving speed of 38.3 km/h (23.8 mph). For the Highway, the dominant term is aero because of the higher average driving speed of 78.1 km/h (48.5 mph). The smallest term is inertia because of the small number of accelerations.

The following is a brief history of each of the parameters affecting the tractive and braking energies:

r_0 : In the mid 1970s, a typical value for bias-belted tires, then the norm, was 0.015. Today radials are the norm, and $r_0=0.006$ is not uncommon. In addition to the change in belt orientation, this reduction has been accomplished by changes in sidewall and tread design, type and amount of material, and higher inflation pressures. However, as r_0 decreases the traction decreases and wear increases, so little further progress is expected.

C_D : In the 1970s, a typical value for American passenger cars was 0.55, with 0.45 being typical in Europe. Today, values for most American and European cars are between 0.35 and 0.30, with some even below 0.30. Further reduction that is production-feasible will probably have to come from reducing underbody drag. The light trucks have higher values, with most falling in the 0.40 to 0.50 range.

⁸ The Midsize Car defined by the vehicle characteristics in Table 2 will be used throughout this paper to provide illustrative values for the various fuel-consumption parameters in Equation (24).

A: People are not getting any smaller, and may be getting bigger. The frontal area of American passenger cars has stayed at typically 2 m². Vehicles in the light-truck category are significantly larger, with most falling in a range of 2.5 to 3.5 m².

M: The vehicle mass has received tremendous attention, and large reductions have been made by:

- Reducing the outside dimensions of vehicles while maintaining interior space
- Redesigning many components to use less material
- Substituting lightweight materials (aluminum, magnesium, polymers).

In a short span of time after the start of fuel economy regulation the industry-average mass of passenger cars was reduced about 25%. It then remained relatively constant, but has begun to increase in recent years as safety regulations have tightened and as customer preferences have shifted. Also, the heavier light-truck category has become more and more prevalent and now represents about 50% of sales. The weight used when making EPA fuel economy measurements is prescribed by a table that translates *loaded weight (curb weight + 300 pounds)* into an equivalent *test weight* (Ref. 2).

(I_w/r_w^2): For 4-wheel vehicles, the total rotating inertia of the wheel assemblies is equivalent to an increase of about 2 to 3% in vehicle mass.

Average Drivetrain Efficiency, $\tilde{\eta}_{dr}$

A second term on the demand side is the energy-transfer-weighted average drivetrain efficiency, $\tilde{\eta}_{dr}$, during powered driving. Automatic transmissions are the norm in the U.S. A schematic of such a drivetrain is shown in Figure 6. The drivetrain (torque converter, chain, gearbox, prop shaft, final drive, and tires) transfers brake energy from the engine down to the road, where its value must be E_{TR} .

The relationship between engine speed, N , and vehicle speed, V , is⁹

$$\frac{N}{V} = \frac{(GR)(FDR)}{(SR)(CR)2\pi r_w} \quad (28)$$

⁹ The ratio parameters in the numerator (GR and FDR) are defined as input rotative speed to output rotative speed, while those in the denominator (SR and CR) are output speed to input speed.

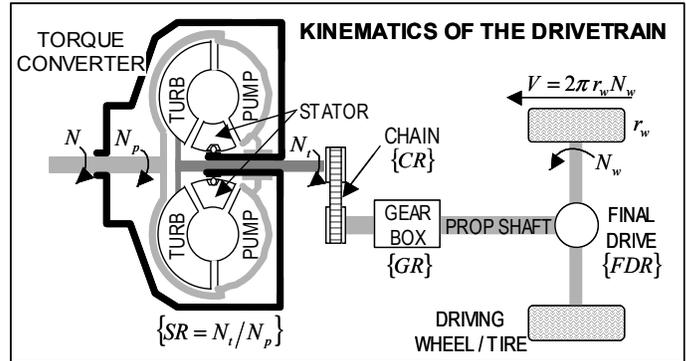


Figure 6. Schematic of a Drivetrain with a Torque Converter.

The primary purpose of the automatic transmission is to free a driver from the task of gear shifting. The fluid connection in the torque converter provides the following additional advantages:

- It isolates the driveline and vehicle from the torsional vibrations of the engine.
- It permits a range of (N/V) in each gear because the ratio of turbine speed to pump speed, SR , can vary. Although not subject to driver control, this is particularly useful in strong vehicle accelerations by permitting SR to be significantly less than unity. This translates to a higher (N/V) in a current gear (see Equation (28)). In first gear this amounts to the effective addition of a lower gear, improving vehicle acceleration from a standstill.
- It permits vehicle creep at zero gas-pedal depression.

The torque converter also has a disadvantage -- significant fluid mechanical losses when SR is not unity. This is particularly the case during accelerations and gear shifts. However, in certain driving situations fluid losses can be reduced by “locking up” the converter so that $SR=1$. This is employed to various degrees in contemporary vehicles.

Representative values¹⁰ of $\tilde{\eta}_{dr}$ for the Midsize Car of Table 2 that is being used as an example are:

- Urban 66%
- Highway 75%

The Urban value is the lower one because of the greater number and magnitude of accelerations, and a greater number of gear shifts.

¹⁰ The illustrative values of $\tilde{\eta}_{dr}$, and of the other fuel-consumption parameters in Equation (24) that will be given later, were determined using a comprehensive vehicle-simulation computer program. They are collected in Table 6, along with a detailed description of the Midsize Car.

Improvement in $\tilde{\eta}_{dr}$ can be accomplished in two primary ways. The first is by providing more torque-converter lockup (referred to as aggressive shift logic). The degree to which it can be used is constrained by the need for customer-acceptable noise, vibration and harshness. A second way is by eliminating the torque converter and automating a manual-shift transmission. Additional but secondary gains can be made by improving the mechanical design of various components of automatic transmissions and by using improved lubricants.

Continuously-variable transmissions are used in some vehicles to better utilize the regions of higher engine efficiency (see the section on $(\tilde{\eta}_b/\eta_{b,max})$). Although effective for that purpose, they have greater mechanical losses than current automatic transmissions, lowering $\tilde{\eta}_{dr}$. This offsets some of the benefit.

Vehicle Accessories, $(E_{b,ACC})^+$

The final term on the demand side is the energy required by vehicle accessories. If this energy is represented by a tare load on the power-steering pump and an electrical load of 10 amps on the generator, representative values of $(E_{b,ACC})^+$ for the Midsize Car are:

- Urban 10% of E_{TR}
- Highway 6% of E_{TR}

Means for reducing this energy requirement include the use of electric power steering, enabling it to be activated only when needed, and a 42-volt electrical system to reduce power losses in wiring and to permit increases in the efficiency of electrically-driven vehicle components.

Maximum Brake Thermal Efficiency, $\eta_{b,max}$

The energy supply side of powered driving is represented by the engine. In Equation (24) its impact on fuel consumption is described by two efficiency parameters, the first of which is the maximum brake thermal efficiency, $\eta_{b,max}$.

Automotive engines operate over broad ranges of speed and load. A representative map of such operation for contemporary gasoline engines is shown in Figure 7.

Brake torque, T_b , is plotted versus engine speed, N , with contours of constant brake thermal efficiency, η_b . In this particular representation, T_b and N are normalized with their values at the maximum-power point (designated by the superscript $*$) and the efficiency contours with the maximum thermal efficiency, $\eta_{b,max}$. A region of highest efficiency is evident. Near the peak, η_b drops off rather slowly; however, in regions

of relatively low load, it decreases fairly rapidly. The first thing that will be addressed is the level of $\eta_{b,max}$.

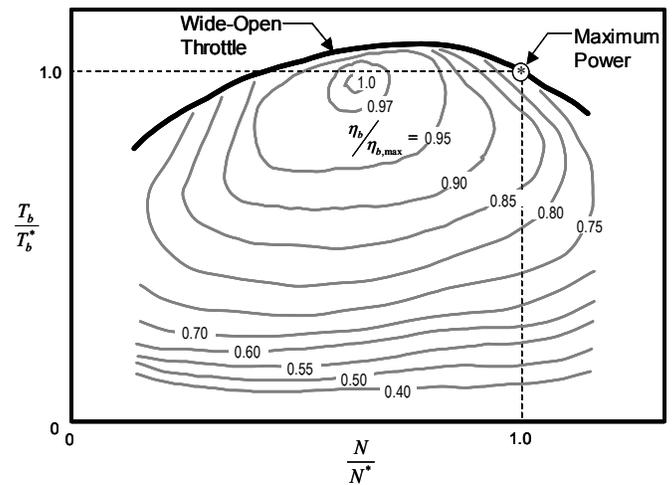


Figure 7. Representative Operating Map for Contemporary Gasoline Engines.

The brake output power of an engine is

$$P_b = P_i - P_p - (P_f + P_a) \tag{29}$$

where P is power, with the components

- b ~ brake
- i ~ indicated
- p ~ pumping (gas exchange)
- f ~ friction
- a ~ auxiliaries essential to independent engine operation

The indicated power, P_i , is the power source. For reciprocating engines operating on a 4-stroke cycle, it is the net gas power exerted on the pistons during the compression and expansion strokes.

Reciprocating engines are batch processors, and P_p is the gas-exchange or pumping power required to ingest the cylinder charge of each batch during the intake stroke of a cycle and to expel the products of combustion during the exhaust stroke.

The friction power, P_f , is that dissipated by frictional forces due to the relative motion of surfaces in sliding and rotating elements of the engine mechanism -- valvetrain, piston and rings, and bearings. It is the consequence of actuating the valves, sealing the reciprocating piston motion, converting linear piston motion to output-shaft rotation with a slider-crank mechanism, and supporting rotating and oscillating components.

The auxiliaries power, P_a , is the subtraction from shaft power required to drive various pumps essential to

independent engine operation (fuel, oil, water), the engine cooling fan, and the generator to produce the electrical energy required for spark ignition and computer control of the engine.¹¹ The convention in the industry is to lump P_f and P_a together and call their sum “friction” power. This obscures the underlying physics. The convention needs to be understood when interpreting reported data on “friction” power.

Dividing Equation (29) by the rate of fuel energy, $\dot{m}_f H_f$, required to produce the power, yields

$$\eta_b = \eta_i - \frac{P_p}{\dot{m}_f H_f} - \frac{(P_f + P_a)}{\dot{m}_f H_f} \quad (30)$$

where η_b is the brake thermal efficiency defined in Equation (10), and the indicated thermal efficiency is

$$\eta_i \equiv \frac{P_i}{\dot{m}_f H_f} \quad (31)$$

The driver of η_b is η_i , and η_i can be decomposed by introducing the following three component efficiencies:

Combustion Efficiency: the fraction of a fuel’s chemical energy that is actually released by combustion,

$$\eta_{comb} \equiv \frac{\dot{Q}_{released}}{\dot{m}_f H_f} \quad (32)$$

Insulation Efficiency: the fraction of released energy that is retained by the working gas, not lost to the walls of the combustion chamber, and therefore added to the thermodynamic cycle,

$$\eta_{insul} \equiv \frac{\dot{Q}_{added}}{\dot{Q}_{released}} \quad (33)$$

Thermodynamic Efficiency: the fraction of retained thermodynamic energy that is converted into indicated piston power,

$$\eta_{therm} \equiv \frac{P_i}{\dot{Q}_{added}} \quad (34)$$

Using these definitions in an expansion of Equation (31)

$$\eta_i = \left(\frac{P_i}{\dot{Q}_{added}} \right) \left(\frac{\dot{Q}_{added}}{\dot{Q}_{released}} \right) \left(\frac{\dot{Q}_{released}}{\dot{m}_f H_f} \right) = \eta_{therm} \eta_{insul} \eta_{comb} \quad (35)$$

The combustion efficiency in this equation is affected by the quality of mixture preparation, and by in-cylinder fluid motions and turbulence.

Insulation efficiency is dependent on the surface-to-volume ratio of the combustion space in the cylinders of an engine, increasing as the ratio decreases. For any given engine displacement, surface-to-volume ratio decreases as the number of cylinders is reduced. If the compression ratio and stroke-to-bore ratio of the cylinders doesn’t change, surface-to-volume ratio is proportional to the cube root of the number of cylinders. For example, going from 8 to 4 cylinders decreases the surface-to-volume ratio by 20.5%, thereby increasing η_{insul} . This efficiency can also be increased by reducing the maximum in-cylinder gas temperature, which can be accomplished by using either lean or dilute mixtures.

Thermodynamic efficiency is the major driver. Its principal variables are identified by the efficiency $(1 - 1/r_c^{\gamma-1})$ of the ideal, constant-volume-heat-addition Otto cycle. While quantitatively overly optimistic for real engines, this expression indicates two variables that have significant impact on the efficiency of those engines. The compression ratio, r_c , is the primary influence, and the larger its value the higher the efficiency for actual engines. However, practical values are constrained by the increasing propensity for engine knock as r_c increases. In contemporary gasoline engines, r_c is about 9.5. The ratio of specific heats, γ , of the in-cylinder gases is also a factor, with larger values giving higher efficiency. Lean mixtures give larger γ .

Efficiency can also be increased by approaching the constant-volume-heat-addition process of the Otto cycle. This requires shorter combustion duration and an appropriate crank angle at which combustion begins (spark timing). The former is dependent on the quality of mixture preparation, and particularly on high values of in-cylinder turbulence. However, very short combustion durations can generate rates of in-cylinder pressure rise that produce excessive engine noise and harshness. Finally, by appropriate valve timing, the ratio of effective expansion stroke to effective compression stroke can be made greater than unity, thereby increasing thermodynamic efficiency.

Although the combustion and insulation efficiencies are high (above 90%), the thermodynamic efficiency is not. Consequently, test data show that their product, η_i , can only be as high as about 37% in good contemporary engines (e.g., spark ignition with port fuel injection, stoichiometric mixtures, and exhaust gas recirculation), and it does not vary greatly with engine speed or load.

The second term in Equation (30) is the pumping power, P_p . Load control in contemporary practice utilizes a throttle to control the engine airflow rate. This generates

¹¹ The extent to which P_a is included in Figure 7 depends on the degree to which the essential auxiliaries are part of the engine configuration used when generating the map.

significant values of P_p at small load fractions. However, as seen in Figure 7, the operating point for maximum η_b is near full load where P_p is not large.

The final term in Equation (30) is the friction and auxiliaries powers, $(P_f + P_a)$. Both P_f and P_a are speed dependent but relatively insensitive to load. Reduced P_f can result from better mechanical design of elements involving relative motion, and from better lubricants. Reduced P_a can be achieved by redesigning auxiliary pumping elements for greater efficiency, and/or by decoupling them from engine speed so that they can be driven (electrically) only at the speed they need to produce their required performance. Furthermore, electrically-driven and thermostatically-controlled engine-cooling fans can be operated only when needed.

In spite of the above avenues for reducing P_p , P_f and P_a , it must be remembered that $\eta_{b,max}$ can only approach its corresponding η_i . It cannot reach or exceed it.

Accounting for these influences on η_b , values of $\eta_{b,max}$ typically range from 32% to 35% for good contemporary gasoline engines. Whether or not the engine map of Figure 7 accounts for the total power requirement of the essential engine auxiliaries depends on the particular configuration of an engine that is used for generating the map. Such data does not usually include all of P_a , and so the value of $\eta_{b,max}$ may be slightly high and the shape and location of the $(\tilde{\eta}_b/\eta_{b,max})$ contours in Figure 7 slightly different.

Diesel engines have an r_c of about 18. Consequently, there are automotive diesels with quoted values of $\eta_{b,max}$ from 40% to 44%. The average is about 25% higher than the average of the previously given values for good contemporary gasoline engines.

Average/Maximum Brake Thermal Efficiency. $\tilde{\eta}_b/\eta_{b,max}$

As seen earlier in Figure 7, engine efficiency η_b varies significantly over the field of an engine's operating map. As a vehicle is driven over an EPA schedule the operating point of its engine moves around on this map during powered driving, and its locus determines the average efficiency $\tilde{\eta}_b$ in accordance with Equation (15).

To facilitate locating this locus on the vehicle's engine map, the plot of Figure 7 can be augmented by adding equilateral hyperbolas representing curves of constant $(P_b/P_b^*) = (T_b/T_b^*)(N/N^*)$, where P_b^* is the maximum engine power. This yields Figure 8.

At any instant during powered driving, the relationship between engine power and tractive power is (see Figure 9),

$$P_b = \frac{P_{TR}}{\eta_{dr}} + P_{b,ACC} \quad (36)$$

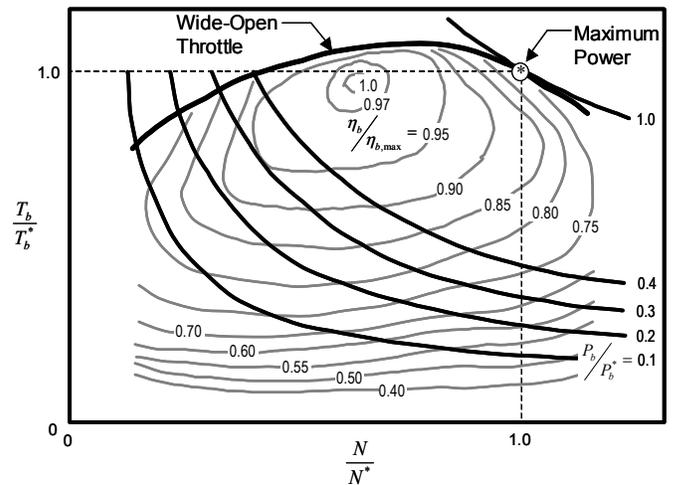


Figure 8. Representative Gasoline-Engine Operating Map with Power Curves.

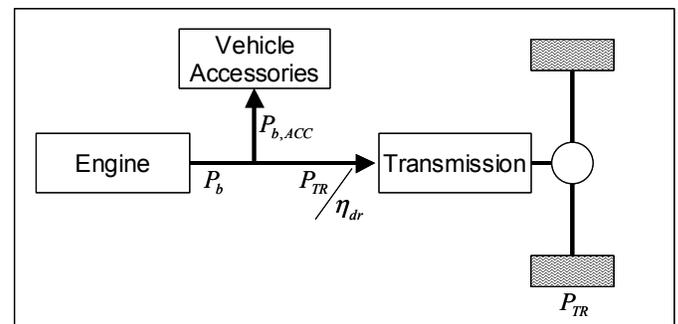


Figure 9. Instantaneous Power-Flow Diagram During Powered Driving

This power corresponds to a particular (P_b/P_b^*) curve in the engine map of Figure 8. The engine power can be produced at any speed on this curve. For each instantaneous vehicle speed, V , the corresponding engine speed, N , is given by Equation (28). For the drivetrain in an existing vehicle, the only selectable parameter in this equation is the gear ratio, GR . It has a different value for each gear. Vehicle engineers select a gear based on considerations such as driveability, fuel economy, acceleration performance, and exhaust emissions. Following this procedure instant by instant throughout the powered-driving segment of a driving schedule, the *detailed trajectory* of instantaneous engine

operating points through the $(\eta_b / \eta_{b,\max})$ contours of the engine map is established.

The *general region* of the map in which this operating locus occurs can be located. As a vehicle is driven over a schedule, the schedule's $V(t)$ dictates the instantaneous tractive power, P_{TR} , that is required, and its value is determined by Equation (25). At some point in the schedule P_{TR} will be a maximum. Typical values of the maximum P_{TR} for a range of vehicle sizes are shown in Table 3.

VEHICLE	URBAN, [kW]		HIGHWAY, [kW]	
	Maximum	Average	Maximum	Average
Small Car	25.5	4.76	21.8	7.75
Midsize Car	34.9	6.32	29.6	9.56
Large Car	37.3	6.79	31.7	10.4
Midsize SUV	43.1	7.73	36.7	12.9

Table 3. Tractive Power Required on the EPA Schedules.

The vehicles are ordered with increasing size and mass from top to bottom. The maximum power increases with vehicle size, and is greater for the Urban than for the Highway schedule. The greater Urban value is the result of the maximum required vehicle acceleration being greater on that schedule.

The time-average P_{TR} for the vehicles during powered driving is also of interest. It is the required tractive energy, E_{TR} , divided by the duration of powered driving, i.e.,

$$\bar{P}_{TR} \equiv \frac{E_{TR}}{\tau} \quad (37)$$

The duration, τ , is not known *a priori* and so needs to be determined. It is another output of the tractive-energy analysis and is shown in Figure 10, where the duration, τ , as a percentage of total schedule time, T , is plotted versus a range of $(C_D A/M)$ and three values of r_0 . (Wheel rotating inertia is neglected in this plot; however, the effect of including it is less than 1%). For both the Urban and Highway schedules the percentage is almost vehicle and tire independent. It is significantly smaller for Urban, averaging slightly less than 60%.

Using Figure 10 and Equations (26) and (37), the average values of P_{TR} shown in Table 3 are generated. The \bar{P}_{TR} values are much smaller than the maximum values. In contrast to the maximum value, \bar{P}_{TR} is smaller on the Urban than on the Highway schedule, a consequence of the average speed of the vehicle while

in motion on the Urban schedule being only about half of that on the Highway.

The ratio of average to maximum tractive power is shown in Figure 11 for the broad range of $(C_D A/M)$ and the three values of r_0 in Figure 10. (Wheel rotating inertia is neglected in this plot; however, the effect of including it is less than 1%). This ratio is about 20% for the Urban schedule, and it ranges between 30% and 45% for the Highway.

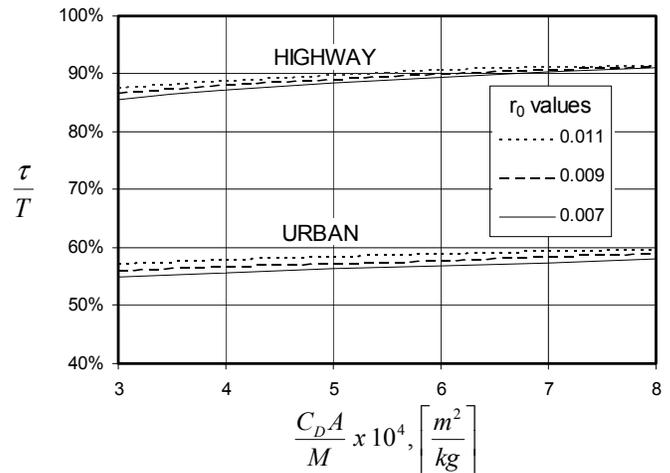


Figure 10. Percentage of Schedule Duration Devoted to Powered Driving.

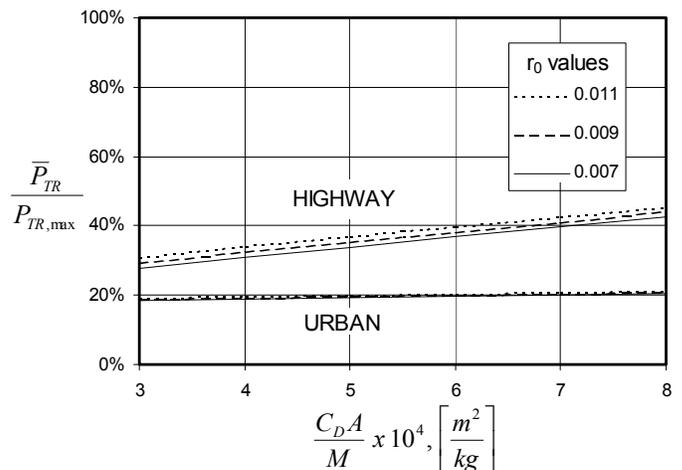


Figure 11. Ratio of Average-to-Maximum Tractive Power.

To relate the tractive powers in Table 3 to the engine map of Figure 8, it is necessary that they be normalized with respect to the engine's maximum (rated) power, P_b^* . Therefore, this power needs to be addressed.

A value of P_b^* equal to $P_{TR,max}$ would not give vehicle performance capability that most drivers would find acceptable. More power is necessary, and its value is primarily determined by the desired performance of the vehicle. Good acceleration is necessary for safe passing of another vehicle, for safe blending into freeway traffic, and for maneuvering in traffic. More power is also needed for hill climbing and for towing.

A common measure of acceleration capability used in the industry is the time, t_{0-60} , required to go from a standing start to 60 mph at wide-open-throttle (WOT). A useful rule-of-thumb for establishing the required engine power rating (in powertrains with an automatic transmission using a torque converter) is

$$P_b^* [\text{kW}] \approx \frac{0.7 M [\text{kg}]}{t_{0-60} [\text{s}]} \quad (38)$$

The larger the M and the shorter the t_{0-60} , the greater the required P_b^* .

In the early 1970s, a typical value of t_{0-60} was 15 s. Today it is 10 s or less. At 10 s for the Midsize Car with $M = 1644$ kg, Equation (38) yields $P_b^* = 115$ kW. This is three to four times greater than the maximum P_{TR} values shown in Table 3 for this car. With this maximum engine power, the tractive-power fractions relevant to the general region of engine operation on the normalized engine map are shown in Table 4 for the Midsize Car.

	URBAN	HIGHWAY
\bar{P}_{TR} / P_b^*	0.05	0.08
$P_{TR,max} / P_b^*$	0.30	0.25

Table 4. Typical Tractive-Power Fractions for a Midsize Car on the EPA Schedules [$r_0=0.009$, $C_D=0.338$, $A=2.06$ m², $M=1644$ kg, $I_w=0.949$ kg m², $r_w=0.320$ m, $t_{0-60}=10$ s, $P_b^*=115$ kW].

Although of interest, the *tractive*-power fractions in Table 4 are not sufficient for identifying the region of operation on the engine map during a driving schedule. This requires the average *brake* power experienced, and the maximum *brake* power required (\bar{P}_b and $P_{b,max}$, respectively), and their magnitudes relative to P_b^* .

The time-averaged *brake* power for powered driving is

$$\bar{P}_b \equiv \frac{1}{\tau} \int_+ P_b dt = \frac{(E_b)^+}{\tau} \quad (39)$$

where the instantaneous values of P_b are given by Equation (36), and $(E_b)^+$ by Equation (17). Although P_{TR} is its dominant, controlling factor, P_b is also influenced by drivetrain efficiency, η_{dr} , and the power required by vehicle accessories, $P_{b,ACC}$. The value of \bar{P}_b relative to P_b^* for the Midsize Car of Table 4 is given in Table 5.

The maximum brake power required during powered driving is $P_{b,max}$. It can only be determined by examining the values of P_b from Equation (36) throughout a driving schedule. It should occur near the point where P_{TR} is a maximum. To find its actual location requires a comprehensive vehicle-simulation capability. Values of $P_{b,max}$ relative to P_b^* for the Midsize Car are shown in Table 5.

	URBAN	HIGHWAY
\bar{P}_b / P_b^*	0.08	0.11
$P_{b,max} / P_b^*$	0.36	0.30

Table 5. Typical Engine Brake-Power Fractions for a Midsize Car on the EPA Schedules [$r_0=0.009$, $C_D=0.338$, $A=2.06$ m², $M=1644$ kg, $I_w=0.949$ kg m², $r_w=0.320$ m, $t_{0-60}=10$ s, $P_b^*=115$ kW].

With the power fractions of Table 5, it can be seen in Figure 8 that average engine operation for both schedules is predominantly in regions of the map where η_b is significantly less than $\eta_{b,max}$, strongly impacting $\tilde{\eta}_b$.

For the Midsize Car with a state-of-the-art gasoline engine, values of $(\tilde{\eta}_b / \eta_{b,max})$ are:

- Urban 68%
- Highway 78%

These values are higher than what might have been expected from Table 5 and Figure 8. A contributing factor is the fuel-consumption-weighted nature of $\tilde{\eta}_b$, Equation (15), a nature that resulted directly from the derivation of the fuel-consumption equation. Operating points having higher fuel rates are at higher engine load fractions; as observed in Figure 8, these have higher η_b and so have the greatest impact on $\tilde{\eta}_b$.

Before proceeding further, another important point can be made. There is a fundamental tradeoff between fuel consumption and vehicle performance. For any given engine technology, the larger the P_b^* to get better vehicle performance, the farther down on the map is engine operation on the EPA schedules; hence, the $\tilde{\eta}_b$

is lower and the fuel consumption higher. Conversely, by accepting a lower level of vehicle performance with a smaller P_b^* , the farther up on the engine map is engine operation and the lower the fuel consumption. Consequently, the fuel-consumption ratings *per se* of vehicles should not be compared without knowledge of the corresponding levels of vehicle performance.

A variety of techniques for increasing $(\tilde{\eta}_b/\eta_{b,\max})$ exist, and these will be described in what follows. The objective of all of them is to get the locus of engine operating points into regions of higher η_b . This can be accomplished by shifting the locus of engine operation, either to lower (N/N^*) or to higher load fractions, or by improving the efficiency-contour topology of the engine map.

Shifting the Locus of Engine Operation

The locus can be shifted by using more gears in discrete-ratio transmissions. Providing more choices of engine speed at any required P_b permits better utilization of the engine map. Four-speed automatic transmissions are the most common, but five-speeds are also prevalent. Even six-speeds are being considered. The additional gears enable overdrive gears that, when added to the previously highest gear, provide a lower minimum value of (N/V) . The corresponding lower N permits engine operation for any required P_b to generally be in a region of higher engine efficiency (see Figure 8).

The ultimate configuration in this scenario is the Continuously Variable Transmission (CVT). It permits engine operation at any speed within its speed-ratio limits. The variable-pulley belt type is the most common; however, it is torque limited and only found in smaller cars.

As seen in Equation (28), (N/V) also depends on the final-drive ratio, FDR . If a gearbox has already been prescribed, FDR is sometimes established by the gradeability desired for a vehicle. Gradeability refers to vehicle operation under prescribed conditions of velocity, % grade, transmission gear, and torque-converter mode. If the WOT torque curve in the low-speed region of an engine map can be raised (e.g., by appropriate valve timing), the required FDR can be reduced. The corresponding reduction in (N/V) provides the same benefit to $(\tilde{\eta}_b/\eta_{b,\max})$ that has been described for an additional high gear in the transmission.

For an existing engine, the locus of its operation can also be shifted by deactivating some of its cylinders. A cylinder is deactivated by disabling its valve motions (with the intake and exhaust valves in their closed positions) and shutting off its fuel supply (which is feasible with port-fuel-injection engines). As cylinders are deactivated, the remaining operating cylinders are forced to higher and more efficient load fractions in order

to maintain any required P_b . This shifting to a curve of higher (P_b/P_b^*) in Figure 8 results in higher values of $(\eta_b/\eta_{b,\max})$, and hence of $(\tilde{\eta}_b/\eta_{b,\max})$.

An alternative is to physically scale an existing engine to a smaller displacement with a lower P_b^* , while providing pressure-charging capability to recover the original P_b^* when needed during off-schedule driving. The reduction in P_b^* pushes engine operation to larger values of (P_b/P_b^*) during the EPA tests. If the scaling is done properly, the normalized full-scale map of the naturally-aspirated engine is largely retained. However, the shape of the WOT torque curve for pressure-charged operation can be much different.

Improving the Efficiency-Contour Topology of an Engine Map

Referring to Figure 8, the geometry of the constant- η_b contours can be described in terms of a geography (the location of the maximum-efficiency island) and a topography (the slope with which η_b falls from its maximum value). Using the analogy of a topographical map, the objective in this scenario is to make the efficiency "hill" as flat as possible by raising its "lowlands" while maintaining its "peak". The flatter the topography, the greater the value of η_b in regions where $\eta_b < \eta_{b,\max}$, thereby increasing $\tilde{\eta}_b$. This also reduces the sensitivity of $\tilde{\eta}_b$ to the locus of engine operating points. Focusing on technologies that improve efficiency in the regions of high fuel consumption experienced during the EPA test schedules would be particularly beneficial to regulated fuel economy.

More-efficient means for controlling air rate can flatten the engine-map topography. Consider an engine operating at WOT at any particular speed. To first order, P_b is determined by the fuel rate, so the fuel rate has to be halved to cut the power to half the WOT value. But conventional gasoline engines need to run at the stoichiometric fuel-air ratio for best performance of the 3-way catalytic converter, and so the air rate must also be cut in half. This is usually accomplished by restricting or "throttling" the intake flow. While being simple, effective and cheap, it has the disadvantage of lowering the intake-manifold pressure, thereby increasing engine pumping work, P_p . Combined with the reduction in fuel rate, this significantly increases $(P_p/\dot{m}_f H_f)$, thereby reducing η_b (see Equation (30)).

Figure 12 is a schematic of a single cylinder of a multicylinder engine. Its air rate is

$$\dot{m}_a = \rho_i \left[\left(\frac{\pi B^2}{4} \right) S_e \right] \left(\frac{N}{n/V} \right) \quad (40)$$

where ρ_i is the air density in the cylinder at the end of intake, S_e the effective intake stroke, and $N/(n/\nu)$ the cycle-rate of engine operation, with n being the number of strokes per engine cycle and ν the number of strokes per engine revolution.

The throttling method of air-rate control reduces \dot{m}_a by reducing ρ_i , which is accomplished by lowering the in-cylinder pressure.

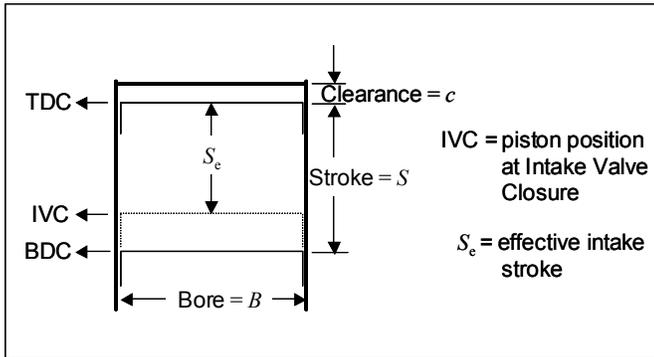


Figure 12. Schematic of an Engine Cylinder.

If continuously variable valve timing is available, the effective intake stroke, S_e , can be reduced either by earlier closing of the intake valve (EIVC) or by late closing (LIVC). In the first case air ingestion is terminated before BDC; in the second one, some ingested air is expelled back into the intake manifold on the upstroke. In the ideal case the in-cylinder pressure is not reduced with load and so pumping work does not increase. S_e can also be reduced by reingesting exhaust products from previous engine cycles, thereby reducing the cylinder volume available for fresh charge. This can be accomplished either by retarding the closing of the exhaust valve or by using exhaust gas recirculation (EGR).

Alternatively, the engine stroke, S , itself can be varied by appropriate mechanical design, and again without changing pumping work. Mechanisms of this type have been invented, but none have proven to be practical.

There is a type of engine in which the air rate at any given speed does not have to be reduced to decrease power, and so pumping work doesn't increase. It is the naturally-aspirated, compression-ignition diesel engine. Ideally, combustion is initiated around each fuel droplet and so the combustion process is not dependent on flame propagation *per se*. Therefore a broad range of overall fuel-air ratio can be tolerated, down to very lean levels. This permits engine power at any speed to be decreased by simply reducing the in-cylinder fuel injection. With minimal and essentially unchanging

pumping work, higher values of η_b are achieved at small load fractions, and this leads to greater values of $(\tilde{\eta}_b/\eta_{b,\max})$ that provide a significant fuel-consumption advantage.

There is also a type of engine that has hybrid load control. It is the direct-injection gasoline engine. In the region (N/N^*) greater than about 0.5, load control is by the same throttling method used for port-injection engines, but early injection is employed in order to have more time for preparation of the nominally homogeneous, stoichiometric charge. However, in the region of (N/N^*) less than about 0.5, this method of load control is only employed down to a particular load fraction. At this point throttling is terminated, and further load reduction achieved by reducing fuel rate at constant air rate. This requires that the fuel be confined to the fraction of in-cylinder air that will produce an approximately stoichiometric mixture. This stratification is facilitated by late injection, and requires careful control of the fuel spray and the in-cylinder air motions. In addition, since there may only be one ignition point and the flame has to propagate throughout the fueled region, the stratified charge has to be located so that it contains the ignition point. The result is less pumping work in this lower range of load fractions, which increases η_b . The region of the engine map for stratified-charge operation is chosen to coincide with that where the engine operates much of the time on the EPA test schedules.

Finally, there is a means of improving the topology that is not air-rate related. The friction and auxiliaries powers, P_f and P_a , in Equation (30) are primarily engine-speed dependent. Even though they have little variation as load is reduced, they become larger and larger fractions of the reduced $\dot{m}_f H_f$ that produces the load reduction. Since indicated efficiency η_i does not vary greatly, this increasing subtraction from it reduces η_b in Equation (30). Efforts to reduce P_f and P_a can therefore improve a map's topology, as well as increase its $\eta_{b,\max}$.

In concluding the consideration of $(\tilde{\eta}_b/\eta_{b,\max})$ it is worth emphasizing that although there are a variety of ways for increasing this ratio and a variety of combinations, their individual effectiveness can be limited by redundancies and interferences between them. There may be a number of ways to "skin the cat," but there is only one cat to skin. For example, the flatter the efficiency-contour topography of an engine map, the less there is to be gained from additional gears in the transmission or from downsizing the engine. Also, as previously indicated for a contemporary Midsize Car, the values of $(\tilde{\eta}_b/\eta_{b,\max})$ are already large, and their upper limit is unity. This limit can only be approached, not reached!

Braking

As indicated in the fuel consumption equation (Equation (24)) the total duration of the braking segments in a schedule is $t_{brkg} = T - \tau - t_{idle}$. With τ given in Figure 10, and T and t_{idle} prescribed, $t_{brkg} \approx 25\%$ of T on the Urban schedule and $\approx 10\%$ on the Highway.

By definition, engine power is not required for vehicle propulsion during this driving mode. Consequently, some degree of fuel-rate reduction is used in contemporary vehicles. It varies from vehicle line to vehicle line, with vehicle driveability being a constraint. For the Midsize Car that has been considered, 12.9% of the fuel is consumed during braking on the Urban schedule and 4.4% on the Highway.

There are efforts to shut the fuel off completely during braking. To be acceptable, the engine has to be restartable in a reliable, quick, and seamless manner. This can be enabled if the starter motor and electric generator are combined into a single unit, the integrated starter generator (ISG). With an ISG, the power available for starting is significantly greater than from a conventional starter motor.

When the engine is not running, the engine-driven vehicle accessories (e.g., power-steering pump, air-conditioning compressor) do not function. This either has to be accepted, or alternative means found for powering them. The latter can be accomplished if they are electrically driven and if there is sufficient energy storage capacity in a battery system to run them throughout the braking periods.

IDLING

The idling time, t_{idle} , is prescribed as 17.6% of T on the Urban schedule and 0.5% on the Highway schedule. Idle fuel rate is determined by engine displacement, idle speed, accessory loads and drivetrain losses. It is constant during idling operation, but engine and vehicle dependent. For the Midsize Car, 6.8% of the fuel is consumed during idle on the Urban schedule and 0.1% on the Highway.

These fuel consumptions can be eliminated by cutting off the fuel. The constraints on doing so, and the means for relaxing them, are the same as those for braking.

If the braking and idling fuel consumptions are combined for the Midsize Car, they represent 19.7% for Urban, and 4.5% for Highway. This makes them obvious and significant targets for efforts to reduce vehicle fuel consumption.

RECAPITULATION

Throughout these discussions, representative values of the fuel consumption parameters for a particular Midsize Car have been presented. For convenience, they are

collected in Table 6, along with a detailed description of the vehicle.

FUEL CONSUMPTION PARAMETERS	UNITS	URBAN	HIGHWAY
E_{TR}	kJ	5,301	6,666
$\tilde{\eta}_{dr}$	%	66.0	74.8
$(E_{b,ACC})^+$	kJ	541	437
$(\tilde{\eta}_b/\eta_{b,max})$	--	0.681	0.782
Powered Time	%	56.9	88.3
Braking Time	%	25.5	11.2
Idle Time	%	17.6	0.5
Powered Fuel	%	80.3	95.5
Braking Fuel	%	12.9	4.4
Idle Fuel	%	6.8	0.1
Schedule Fuel Economy	mpg	19.5	33.6
Combined Fuel Economy	mpg	24.0	

VEHICLE & SUBSYSTEM PARAMETERS

VEHICLE	$M=1644$ kg; $A=2.06$ m ² ; $C_D=0.338$; $r_\theta=0.009$; $I_w=0.949$ kg m ² ; $r_w=0.32$ m
ENGINE	$P_b^*=110.9$ kW; $\eta_{b,max}=34.5\%$
DRIVETRAIN	Torque Converter; 4-Speed Gearbox; Two-Wheel Drive
PERFORMANCE	$t_{0-60}=10.4$ s

Table 6. Fuel-Consumption-Parameter Breakdown for a Midsize Passenger Car.

THE NRC REPORT

In legislation for fiscal year 2001, the U.S. Congress requested that the National Academy of Sciences, in consultation with the Department of Transportation, conduct a study to evaluate the effectiveness and impact of CAFE standards. In response, the National Research Council (NRC) established the Committee on *Effectiveness and Impact of Corporate Average Fuel Economy (CAFE) Standards* (Ref. 4). The Committee began its work in early February 2001. A prepublication report was released in July 2001, and the final report in January 2002. It utilizes a methodology for estimating the impact of a variety of engine, transmission and vehicle technologies on vehicle fuel economy. Its methodology will be critically examined so that it can be compared to what has been developed in the present paper.

As in the present paper, the NRC methodology is based on fuel consumption. It has the following description.¹² *“For each technology considered, the tables give an estimated range for incremental reductions in fuel consumption.” “The analysis ... is based on the average fuel consumption improvement For each vehicle class the average fuel consumption improvement for the first technology selected is multiplied by the baseline fuel consumption. This is then multiplied by the average*

¹² Italicized quotations in this section of the paper represent statements extracted directly from the NRC report.

improvement of the next technology, etc.” No rationale or physical evidence is offered to support this procedure, nor are any conditions or limitations placed on it.

Analysis of the NRC methodology’s attributes is facilitated by formalizing the preceding description in functional form. That is,

$$g_n = g_0 \left[1 - \frac{(\%)_1}{100} \right] \left[1 - \frac{(\%)_2}{100} \right] \dots \left[1 - \frac{(\%)_i}{100} \right] \dots \left[1 - \frac{(\%)_n}{100} \right] \quad (41)$$

where g_0 is the fuel consumption of a baseline vehicle, n the total number of technologies applied to the baseline vehicle, $(\%)_i$ the average incremental percentage reduction in fuel consumption provided by technology i , and g_n the fuel consumption of the vehicle after the application of all n technologies.

Although the basis for such a formulation is not given, the following is a possible rationalization. Considering a scenario in which a group of n technologies is applied to a baseline vehicle, the application of the first technology reduces fuel consumption to

$$g_1 \equiv g_0 \left[1 - \left(\frac{g_0 - g_1}{g_0} \right) \right] \equiv g_0 \left[1 - \frac{(\%)_1}{100} \right] \quad (42)$$

Similarly, applying the second technology,

$$g_2 \equiv g_1 \left[1 - \frac{(\%)_2}{100} \right] \quad (43)$$

If the second technology does not affect the fuel-consumption impact of the first technology, and vice versa, then g_1 in Equation (43) is given by Equation (42), yielding

$$g_2 = g_0 \left[1 - \frac{(\%)_1}{100} \right] \left[1 - \frac{(\%)_2}{100} \right] \quad (44)$$

If this procedure is continued to the application of the n th technology, with the fuel-consumption effect of each technology being assumed independent of the others, the result is Equation (41). All of the physics of fuel consumption have to be captured in the numerical values used for the $(\%)_i$. Also, these values are generally not independent of one another and they depend on the vehicle system to which they are applied. The method for assigning them therefore needs to be scrutinized.

The values of $(\%)_i$ used in the Report are compiled in Table 7. They relate to combined fuel consumption. The first column contains the value for each technology when it is the only technology applied to a baseline vehicle.

TECHNOLOGIES			FUEL-CONSUMPTION IMPROVEMENT (%)	
			From Base	From Ref. ¹
ENGINE	Production Intent	Friction Reduction	1 - 5	1 - 5
		Low-Friction Lubricants	1	1
		4-Valve vs 2-Valve	2 - 5	2 - 5
		Variable Valve Timing	4 - 8	2 - 3
		Variable Valve Lift & Timing	5 - 10	1 - 2
		Cylinder Deactivation	8 - 16	3 - 6
		Accessory Improvement	3 - 7	1 - 2
		Supercharging/Downsizing	7 - 12	5 - 7
	Emerging	Intake-Valve Throttling	8 - 16	3 - 6
		Camless Valve Actuation	10 - 20	5 - 10
Variable Compression Ratio		9 - 18	2 - 6	
TRANSMISSION	Production Intent	5-Speed Automatic	2 - 3	2 - 3
		Continuously Variable (CVT)	6 - 11	4 - 8
		Aggressive Shift Logic	3 - 6	1 - 3
		6-Speed Automatic	3 - 5	1 - 2
	Emerging	Automated-Shift Manual	6 - 10	3 - 5
		Advanced CVT	6 - 13	0 - 2
VEHICLE	Production Intent	Aero Drag Reduction	1 - 2	1 - 2
		Lower Rolling Resistance	1 - 1.5	1 - 1.5
	Emerging	42-Volt Electrical	3 - 7	1 - 2
		Integrated Starter/Generator	6 - 12	4 - 7
		Electric Power Steering	3.5 - 7.5	1.5 - 2.5
		Weight Reduction (5%)	3 - 4	3 - 4

¹ "Reference refers to a vehicle with prior technologies already implemented. Thus it is the incremental improvement in a series of steps. It is lower than the base improvement (except for the first step in each category) to account for double-counting and other diminishing returns."

Table 7. Fuel Consumption Benefits of Various Technologies (Extracted from the NRC Report)

These values were derived from a variety of sources. As quoted in the Report, *“The analysis was complicated by the need to infer potential fuel consumption benefits from published data in which experimental results were based on European (NEDC) or Japanese (10/11 mode) test cycles.”* Furthermore, as noted in Reference 5, *“In many of the technology studies cited in the Report, fuel consumption reduction projections are taken from individual engine test points rather than from EPA driving-cycle results. ... EPA unadjusted combined fuel consumption results are rarely quoted in the cited studies (at best, only 15 of the 44 cited studies produce results for the appropriate EPA driving cycles). Consequently, the quality of the estimates ... are diminished by the use of information that is not representative of fuel consumption on the EPA driving cycles”* Clearly, there is considerable uncertainty about the $(\%)_i$ values in the first (From Base) column.

The second column of Table 7 has *incremental improvements* for the technologies that are generally smaller than those in the first column. The rationale for the reductions is stated at the bottom of the Table, and taken verbatim from the Report. *“Reference refers to a vehicle with prior technologies already implemented. Thus it is the incremental improvement in a series of steps. It is lower than the base improvement (except for the first step in each category) to account for double-counting and other diminishing returns.”* As for the method used for quantifying the reductions, the Report states: *“The committee notes that its analysis of the incremental benefits of employing additional technologies was, of necessity, based largely on engineering judgment.”* It *“acknowledges that, although it was conservative in its estimates of potential gains attributable to individual technologies (in an attempt to account for potential double-counting) some overestimation of aggregated benefits ... may have occurred”* *“As technologies are added, the overall uncertainty increases.”*

The preceding declarations indicate that the NRC methodology has shortcomings, and these will be delineated. First of all, the relevant scenario is one in which groups of technologies are applied to a vehicle system, and fuel consumption on a driving schedule only assessed after an entire group has been applied. The sequential order in which they are applied is therefore irrelevant. To identify one of the technologies as the first one applied, and therefore entitled to the generally larger $(\%)_i$ value in the “From Base” column in Table 7, is not justified.

Second, the various technologies in a group are not necessarily independent of each other. The $(\%)_i$ value of a technology depends on the nature of the other technologies with which it is grouped, and on the vehicle system to which the group is applied. For example, the $(\%)_i$ value for a CVT will be less when it is grouped with

engine technologies that reduce pumping work in order to flatten the η_b topography of the engine map, than when grouped with technologies that reduce the vehicle energy requirement but do not affect the engine map. Consequently, using a single value of $(\%)_i$ for a technology is not realistic even for a single vehicle system. It is even more unrealistic when applied to all vehicle systems, since ten different vehicle classes are considered in the NRC Report, ranging in size from subcompact passenger cars to large pickup trucks.

Third, the simple arithmetic average of the upper and lower values for the range of $(\%)_i$ for each technology is used in Equation (41) without any statistical substantiation for doing so.

There is an additional comment on the overall methodology. Equation (41) can relate to changes in fuel consumption for any driving schedule. However, the methodology applies it to combined fuel consumption even though there is no combined schedule *per se*. Because the Urban and Highway driving schedules are fundamentally different, the change in fuel consumption resulting from the application of a technology can be very different for them, the magnitude of the difference depending on the vehicle system. Therefore, a separate fuel-consumption evaluation should be made for each schedule, and the results then combined to obtain the value subject to regulation.

Attachment C of Appendix F of the Report states the following objective: *“In an attempt to determine whether some fundamental flaws, resulting in gross errors, had inadvertently entered the judgment-simplified analysis described above, the committee conducted a simulation of a single vehicle (midsize SUV), for which it had access to data that could be used to attempt a more in-depth energy consumption/balance-type analysis.”* Although the NRC methodology works only with combined fuel consumption, this inadequately defined simulation produced a set of Urban, Highway, and Combined fuel economies for each of five different technology scenarios. Close examination of the resultant tabular data for the various scenarios considered reveals that combined fuel economy was determined as

$$m'_C = 0.55\tilde{m}_U + 0.45\tilde{m}_H \quad (45)$$

i.e., by applying the 55/45 weighting to fuel economy rather than fuel consumption (see Equation(4)). This produces values of combined fuel economy that are erroneously high, the error increasing as $(\tilde{m}_H / \tilde{m}_U)$ increases above unity. The smallest and largest values of $(\tilde{m}_H / \tilde{m}_U)$ in Attachment C are 1.38 and 1.78, respectively, with corresponding values of m'_C that are 2.6% and 8.5% greater than the correct values.

TECHNOLOGY BENEFITS			FUEL-CONSUMPTION PARAMETERS						
			E_{TR}	$\tilde{\eta}_{dr}$	$\eta_{b,max}$	$\tilde{\eta}_b/\eta_{b,max}$	$(E_{b,ACC})^+$	$(\bar{m}_f)_{brkg}$	$(\bar{m}_f)_{idle}$
■ Primary Benefit □ Secondary Benefit ▽ Secondary Penalty									
ENGINE	Production Intent	Friction Reduction			□	■			
		Low-Friction Lubricants		■	□	■			
		4-Valve vs 2-Valve			■	□			
		Variable Valve Timing				■		□	□
		Variable Valve Lift & Timing				■		□	
		Cylinder Deactivation				■		□	
		Accessory Improvement			□	■			
		Supercharging/Downsizing				■			
	Emerging	Intake-Valve Throttling				■		□	□
		Camless Valve Actuation			□	■	▽	□	□
Variable Compression Ratio				□	■		□	□	
TRANSMISSION	Production Intent	5-Speed Automatic		▽		■			
		Continuously Variable (CVT)		▽		■			
		Aggressive Shift Logic		■					
		6-Speed Automatic		▽		■			
	Emerging	Automated-Shift Manual		■					
Advanced CVT			▽		■				
VEHICLE	Production Intent	Aero Drag Reduction	■	□					
		Lower Rolling Resistance	■	□					
	Emerging	42-Volt Electrical					■		
		Integrated Starter/Generator						■	■
		Electric Power Steering					■		
Weight Reduction	■	□							

Table 8. Assessment of Technology Benefits Relative to Fuel-Consumption Parameters

The present authors have correlated the technologies of Table 7 with the parameters in Equation (24) of this paper. The result is shown in Table 8, indicating primary and secondary benefits as well as secondary penalties. Its predominant feature is the large number of technologies, 14, for primarily improving $(\tilde{\eta}_b/\eta_{b,max})$. Since that parameter cannot exceed unity, it is unlikely that these technologies can represent independent effects.

The Report contains only one detailed example of its methodology. The subject vehicle is a midsize SUV with a baseline combined fuel economy of 21.0 mpg. A detailed description of the vehicle is not given, and is not required for the methodology. However, such a description is necessary if the predictive accuracy of the methodology is to be evaluated. To permit such an assessment, the contemporary Midsize SUV described in the lower part of Table 9 will be used as the baseline.

For this vehicle, a vehicle-simulation program provided values for all the parameters in fuel-consumption Equation (24), and those relevant to the present assessment are shown in the upper part of the Baseline SUV column of Table 9.

Of the *three successively more aggressive product development paths* considered, the example is for the first level of fuel economy improvement, Path 1. The first step in this Path is an increase of 5% in vehicle mass to accommodate future safety-enhancing features. Using the average fuel-consumption-improvement percentage for weight reduction from Table 7, the NRC methodology predicts an increased fuel consumption of,

$$\begin{aligned} (g_c)_{5\% \text{ mass}} &= (g_c)_{\text{baseline}} * 1.035 = \left(\frac{1}{21.0}\right) * 1.035 \\ &= 0.0493 \text{ gal/mile} \end{aligned} \quad (46)$$

corresponding to 20.3 mpg.

Using Equation (26) to determine the increased E_{TR} on the Urban and Highway schedules due to the increased mass, and putting them into the fuel-consumption Equation (24) for each schedule, the resultant combined fuel economy is 20.5 mpg, if $(\tilde{\eta}_b/\eta_{b,max})$ doesn't change. This represents a 2.6% increase in fuel consumption rather than the NRC-projected 3.5%.

PARAMETERS		BASELINE SUV		+5% MASS		6 ENG + 1 TRANS TECHNOLOGIES	
		URBAN	HIGHWAY	URBAN	HIGHWAY	URBAN	HIGHWAY
NRC	m_c [mpg]	21.0		20.3		24.3	
EQUATION (24)	E_{TR} [kJ]	7140	9390	7420	9580	7420	9580
	$\eta_{b,max}$	0.36		0.36		0.37	
	$\tilde{\eta}_b / \eta_{b,max}$	0.79	0.88	0.79	0.88	0.97	0.98
	\tilde{m} [mpg]	18.0	26.5	17.4	26.0	21.2	29.7
	m_c [mpg]	21.0*		20.5		24.3*	

* Fuel economies intentionally matched to corresponding NRC values

CONTEMPORARY MIDSIZE SUV (BASELINE)

VEHICLE	$M=2268$ kg; $A=2.90$ m ² ; $C_D=0.400$; $r_0=0.0070$; $I_w=2.0$ kg m ² ; $r_w=0.362$ m
ENGINE	$P_b^*=200$ kW; $\eta_{b,max}=36\%$; $(\dot{m}_f)_{brk}=0.314$ gm/s; $(\dot{m}_f)_{idle}=0.310$ gm/s
DRIVETRAIN	Torque Converter; 4-speed gearbox; 4-wheel drive; $\tilde{\eta}_{dr}=0.635_U$ & 0.687_H
ACCESSORIES	$(E_{b,ACC})^+=700_U$ kJ & 600_H kJ
PERFORMANCE	$t_{0-60}=8.0$ s

Table 9. Assessment of NRC Example.

The example then applies six engine technologies (engine friction reduction, low-friction lubricants, 4-valve OHC, variable valve timing, cylinder deactivation, engine accessory improvement), and one transmission technology (5-speed automatic). The product of their individual average¹³ values of $[1-(\%)_i/100]$ from the “From Ref.” Column in Table 7 is 0.833. The NRC-predicted fuel consumption for them is then

$$\begin{aligned} (g_c)_{7\ techs} &= (g_c)_{5\% mass} * 0.833 \\ &= (0.0493) * 0.833 = 0.0411 \text{ gal / mile} \end{aligned} \quad (47)$$

corresponding to $m_c = 24.3$ mpg.

These seven technologies affect $(\tilde{\eta}_b / \eta_{b,max})$ primarily, and $\eta_{b,max}$ secondarily (see Table 8). Assuming that $\eta_{b,max}$ increases to 37% as a secondary benefit, pairs of Urban and Highway $(\tilde{\eta}_b / \eta_{b,max})$ that will produce $m_c = 24.3$ mpg can be determined. The particular pair shown in Table 9 is 0.97 for Urban and 0.98 for Highway. Even for other pairs, however, values of $(\tilde{\eta}_b / \eta_{b,max})$ near unity are required for both schedules. Such values are not realistic. Even if they were, there is then almost no room left for further improvement from the additional engine and transmission technologies that the Report considers for Paths 2 and 3. For example, if $(\tilde{\eta}_b / \eta_{b,max})=1$ on both schedules, the combined fuel economy of the Midsize SUV with the 7 technologies would only increase to 24.9 mpg (i.e., only 2.5% additional opportunity remains). Clearly, the near-unity

values of $(\tilde{\eta}_b / \eta_{b,max})$ in this example indicate that redundancies and interferences are not accounted for in the NRC methodology.

The Report’s overall assessment of its fuel-consumption analysis is as follows: “*The committee’s methodology is admittedly simplistic. Nevertheless, the committee believes it to be sufficiently accurate for the purposes of this study.*” In light of the preceding discussions, this conclusion needs to be reassessed.

UNCONVENTIONAL POWERTRAIN

The analysis of vehicle fuel consumption that has been presented in this paper pertains to the conventional type of powertrain used in contemporary vehicles. However, increasing pressure for large increases in fuel economy is forcing consideration of new types of powertrain. The rationale behind, and nature of, these alternatives will therefore be addressed using the methodology that has been presented.

REGENERATIVE BRAKING

During the EPA driving schedules, particularly the Urban, there are numerous periods during which the required vehicle deceleration is large enough that the continuously acting and inherent retarding forces of aerodynamic drag and tire rolling resistance are insufficient to generate it. Therefore, an additional vehicle-braking force is required. With the gas pedal released during these periods, the powertrain resists being driven by vehicle inertia, and the reaction to this is a negative torque on the driving wheels. This is commonly called “engine braking.” If this still doesn’t produce the required deceleration, the driver must then use the brake pedal to produce an additional negative torque on the wheels, usually on all four.

¹³ An exception is the value of $(\%)_i$ for a 5-speed transmission. The value used by the NRC in the example is 2.0 rather than the Table 7 average of 2.5, so that is what is used here.

The tractive-energy analysis produced an equation (Equation (27)) for the total amount of vehicle energy that must be removed by powertrain and wheel braking over a complete driving schedule. Its magnitude relative to the tractive energy is shown in Figure 13 for a range of vehicle characteristics. (Although wheel rotating inertia is neglected in this plot, the effect of including it is less than 1%).

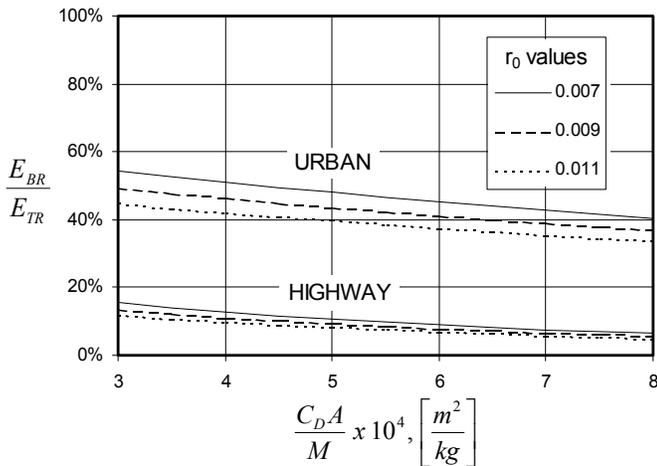


Figure 13. Ratio of Braking to Tractive Energy.

On the Urban schedule, E_{BR} is very substantial. Currently, this energy is dissipated. If some of it could be recovered and stored as useful energy in a suitable storage system it could be used at later times, when propulsion is required, to provide part of E_{TR} (or of $(E_{b,ACC})^+$ if accessories are required) and thereby reduce fuel consumption.

The recovery of braking energy could be accomplished in conventional vehicles by connecting electric motor/generators to each of the non-driving wheels. (The selection of the non-driving wheels is not essential to the concept, but it makes understanding easier). When wheel braking is required the electrical machines would be operated as generators, converting kinetic energy removed from the vehicle into electrical energy. This energy would be stored in a battery system of suitable capacity. At later times when vehicle propulsion is required, energy would be retrieved from storage and used by the electrical machines operating as motors to provide some of the required tractive force.

The impact of regenerative braking on fuel consumption would be captured by incorporating an additional term in the fuel-consumption equation,

$$\tilde{g} = \frac{1}{S\rho_f} \left\{ \frac{\left[\frac{(E_{TR} - \xi_{rg} E_{BR})}{\tilde{\eta}_{dr}} + (E_{b,ACC})^+ \right]}{H_f \eta_{b,max} \left(\frac{\tilde{\eta}_b}{\eta_{b,max}} \right)} + (\dot{m}_f)_{brkg} (T - \tau - t_{idle}) + (\dot{m}_f)_{idle} t_{idle} \right\} \quad (48)$$

where ξ_{rg} is the overall effectiveness of the regeneration process.

Implementation of regenerative braking would require a component in addition to the motor/generators and energy-storage system. The electrical machines are usually AC but the storage system is DC. Consequently AC-to-DC conversion is required on the energy recovery leg, and DC-to-AC on the energy retrieval leg. These two functions can be combined in a single device called an inverter¹⁴.

The effectiveness ξ_{rg} would be influenced by the efficiencies of the electric motor/generators, the AC-to-DC and DC-to-AC conversion processes, the battery storage and retrieval processes, and by the capability for controlling these processes. Also, it would be diminished whenever the rate at which electrical energy could be stored is less than that at which it was being generated, since some regenerated energy would then have to be dissipated. With so many factors involved, high values of ξ_{rg} would not be easy to achieve.

Some of E_{BR} is powertrain braking, so matters could be helped if this braking were reduced so that more of E_{BR} would relate to the wheel braking that is available for regeneration. This could be achieved by deactivating some engine cylinders during braking.

HYBRID POWERTRAINS

The hybrid powertrain represents a progression from such a modified contemporary vehicle with regenerative braking. To facilitate an understanding of the essential opportunities that hybrids provide, a series configuration will be used and is shown in Figure 14.

The engine is uncoupled from the driving wheels and coupled instead to an AC electric motor/generator and an inverter. Engine output is thereby converted to DC electrical energy that is stored in a battery system. This stored energy can be used by this inverter-motor/generator system to start the engine.

¹⁴ Strictly speaking, an inverter is a DC-to-AC converter, and a rectifier an AC-to-DC converter. However, when both conversions are accomplished in a single device, it is commonly called an inverter.

Vehicle propulsion is provided by inverter-motor/generators connected to the driving wheels that receive their input energy from the battery system. These same machines are also used for regenerative braking.

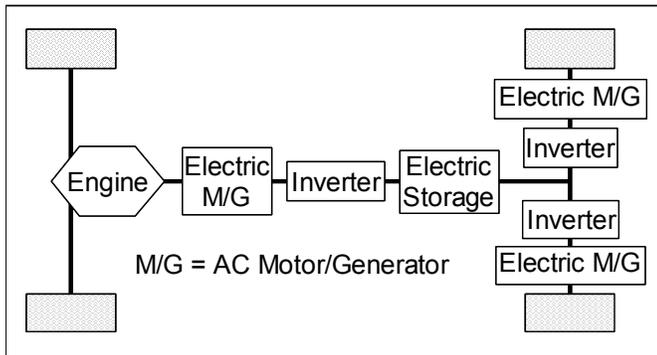


Figure 14. Schematic of a Series Hybrid Powertrain with Electrical Machines

Looking at the fuel consumption equation (Equation (48)), the following opportunities exist:

1. Since the engine is uncoupled from the driving wheels, its output does not have to equal the vehicle's propulsion requirement at every instant. Consequently, whether it is running or not, and what its operating point should be if running, are both independent choices. An ideal scenario would be to run the engine only at its maximum efficiency, making $(\tilde{\eta}_b/\eta_{b,\max})=1.0$, and only for the length of time required to produce the total energy in the numerator of the powered-driving term.
2. Employing regenerative braking ($\xi_{rg} > 0$) reduces the energy that must be supplied by the engine during powered driving.
3. Eliminating engine operation during braking and idling, thereby reducing fuel consumption for these modes to zero.
4. Driving the vehicle accessories electrically and independently of engine operation enables them to be operated only when necessary, and at their most efficient conditions, thereby reducing $(E_{b,ACC})^+$.
5. Replacing the mechanical drivetrain with an electrical one might permit an increase in $\tilde{\eta}_{dr}$.

As indicated previously, these discussions are not intended to suggest that series hybridization is the best configuration. A variety of configurations are being considered by the different automobile companies, and it is not the intention or purpose of this particular paper to discuss or make judgments about them.

CLOSING COMMENTS

The overall fuel consumption of a motor vehicle driving a schedule in which its speed varies with time is captured by Equation (24). It has three components, corresponding to the three possible modes of vehicle operation: 1) *powered driving*, in which the required tractive force at the tire/road interface of the driving wheels is positive; 2) *braking*, in which the required tractive force at these wheels is negative; 3) *idling*, in which the vehicle is stationary and the tractive force is zero. Fuel consumption in each mode is represented by an integral term capturing the relevant controlling physics.

The principal term in Equation (24) is obviously powered driving. It is formulated in terms of brake-energy demand of the vehicle and brake-energy supply by the engine. The demand part is the numerator; it is controlled by the total amount of tractive energy required. This energy is determined by the tire rolling-resistance coefficient (r_0), aerodynamic drag coefficient (C_D), frontal area (A), vehicle mass (M), and wheel-assembly rotating inertia. Its magnitude for the EPA driving schedules is given by the linear Equation (26) with the numerical coefficients in Table 1. The equation accurately represents the benefits that will result from reductions in r_0 , C_D , A , M , or wheel-assembly rotating inertia. The energy-transfer-weighted average driveline efficiency ($\tilde{\eta}_{dr}$) and the brake energy of the vehicle accessories $(E_{b,ACC})^+$ also impact the total brake energy required from the engine.

The brake-energy supply by the engine (per unit mass of fuel) is the denominator; it contains two engine-efficiency parameters. The first is the maximum brake thermal efficiency of the engine $\eta_{b,\max}$. At any engine operating condition, η_b must be less than the corresponding indicated thermal efficiency η_i . For contemporary gasoline engines, η_i is not greatly dependent on engine speed and load. Its maximum value is primarily determined by an engine's compression ratio, and may be as high as 37%. There are three reasons for η_b being less than η_i . First, being a batch processor, gas-exchange (pumping) work is required to ingest the combustible charge each engine cycle and to discharge its products of combustion. In contemporary throttled engines this pumping work increases as load is reduced at any engine speed. Second, there are energy losses due to mechanical friction in various parts of the engine mechanism. Third, energy is required by the various auxiliaries essential to independent engine operation (pumps for fuel, oil and cooling water; fuel injectors; electric generator). Even though considerable effort is devoted to reducing these three subtractions from η_i , particularly pumping work, the $\eta_{b,\max}$ of contemporary gasoline engines does not typically exceed 35%.

The other engine-efficiency parameter is $\tilde{\eta}_b$, the fuel-rate-weighted average η_b over the locus of engine operating points experienced during powered driving. It is less than $\eta_{b,max}$, generally being smaller for the Urban than the Highway schedule because of a greater percentage of engine operation at lower load fractions, where η_b is reduced. Its value is influenced by the ratio of the average brake engine power required, \bar{P}_b , to the rated brake engine power, P_b^* , increasing as (\bar{P}_b/P_b^*) increases. The power P_b^* is a determinant of vehicle performance, i.e., acceleration, hill-climbing, and towing capabilities. Hence, $\tilde{\eta}_b$ can be increased by reducing P_b^* and vehicle performance. It can also be increased by using a transmission with greater gear-ratio selectability or by improving part-load η_b .

The fuel consumption advantages of the diesel engine bear repeating. They are threefold: 1) higher $\eta_{b,max}$ because of a much higher compression ratio than spark-ignition engines, 2) higher $(\tilde{\eta}_b/\eta_{b,max})$ because of non-throttling load control, and 3) higher volumetric fuel economy because of the 15% greater fuel density than gasoline.

By definition, fuel consumed during braking and idling does not contribute to vehicle propulsion. In principle, it can be reduced to zero for fuel-injected engines. However, engine restartability and the operation of vehicle accessories during these modes have to be resolved.

Representative parameter breakdowns for the Urban and Highway driving schedules are shown in Table 6. The particular vehicle is the Midsize Car that has been referenced throughout this paper and whose vehicle and subsystem parameters are defined in the Table.

Equation (24) is most valuable when used in conjunction with a comprehensive vehicle-simulation program that, in addition to overall fuel consumption for the EPA schedules, provides values for the various integral parameters in the equation. This breakdown consolidates the physics, thereby facilitating an understanding of the means by which the particular fuel consumptions are achieved. This knowledge is also useful for identifying promising areas for fuel-consumption reduction.

The parameter breakdown can help reveal interactions that might occur between technologies. For example, if any one of the vehicle variables (r_0 , C_D , A , M) is reduced for an existing vehicle, the required tractive energy is reduced. This reduction in demand tends to reduce fuel consumption. However, the energy reduction causes the engine to operate at smaller load fractions where η_b is less, and so $(\tilde{\eta}_b/\eta_{b,max})$ decreases and offsets part of the benefit of the reduced E_{TR} . This undesired consequence can be largely avoided if

appropriate measures are taken. In the case of reductions in r_0 , C_D , and A , much of the offset in $(\tilde{\eta}_b/\eta_{b,max})$ can be recovered by some appropriate re-gearing of the transmission -- and without sacrificing acceleration performance of the vehicle. In the case of reductions in M , the engine can be appropriately downsized to increase its average operating load fraction -- and again without sacrificing vehicle performance. In both cases, the full potential benefit of the parameter reductions can be realized only after understanding and accounting for the interactions.

Even though the tractive energy of vehicle configurations can be predicted, the fuel-consumption equation itself is not predictive in the same sense. However, it can be made estimative if suitable correlations are developed for its various integral parameters. As has been indicated, values for these parameters can be obtained from a suitable vehicle-simulation program. In order to be realistic, the simulations should observe real-world constraints such as: driveability, as influenced by the shift and torque-converter-lockup schedules; gradeability; adequate launch on acceleration from a standstill; appropriate engine idle speed.

Examples of useful polynomial correlations that have been developed by the authors for passenger cars using vehicle-simulation results are: $\tilde{\eta}_b/\eta_{b,max}$ vs. t_{0-60} ; $\tilde{\eta}_{dr}$ vs. N/V in locked high gear; $(E_{b,ACC})^+$ vs. N/V in locked high gear; $(\dot{m}_f)_{idle}$ vs. Displacement $\times N_{idle}$; $(\dot{m}_f)_{brkg}$ vs. Displacement. With such empirical correlations, the fuel-consumption equation can be transformed into a computer-spreadsheet tool for making rapid and reasonably accurate explorations of opportunities and strategies for significantly reducing the fuel consumption of any baseline vehicle for which the parameters have been evaluated. The physics captured in each term helps suggest how its magnitude might be changed, and how much change might be possible, thereby helping to establish scenarios for fuel-consumption reduction. Once attractive possibilities have been identified, they can then be evaluated and their fuel-consumption benefits quantified using a vehicle-simulation program.

Even without such correlations the equation can be used to explore means for significantly reducing fuel consumption. By applying adjustable multipliers to its various parameters, the individual impact of any parameter on a vehicle's fuel consumption can be assessed ("imagineering"). In this way, key enablers can be identified and R&D efforts then focused on finding ways for achieving their multipliers.

Finally, by presenting results in clear, simple, quantitative terms, the authors hope that this paper will contribute to better understanding of automotive fuel economy, and hence to meaningful discussion and debate on the prospects and possibilities for significant fuel economy improvement.

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NOMENCLATURE

A	vehicle frontal area, [m ²]
b	brake specific fuel consumption of an engine, [kg/MJ]
b_{\min}	minimum brake specific fuel consumption, [kg/MJ]
\tilde{b}	brake-energy-weighted average brake specific fuel consumption over the powered segment of a driving schedule, [kg/MJ]
B	engine cylinder bore, [mm]
BDC	Bottom-Dead-Center piston position
c	piston-to-cylinder head clearance, [mm]
$CAFE$	Corporate Average Fuel Economy
C_D	aerodynamic drag coefficient

CR	Chain Ratio of the transmission
CVT	Continuously Variable Transmission
D	aerodynamic drag force, [N]
E_b	engine brake energy, [kJ]
$(E_b)^+$	total engine brake energy during powered driving, [kJ]
$(E_{b,ACC})^+$	total accessory brake energy during powered driving, [kJ]
$(E_{b,TR})^+$	total engine brake energy delivered to the drivetrain during powered driving, [kJ]
E_{TR}	total tractive energy required at the tire-road interface during powered driving, [kJ]
EGR	Exhaust Gas Recirculation
$EIVC$	Early Intake Valve Closing
EPA	Environmental Protection Agency
F_{TR}	instantaneous tractive force at the tire-road interface, [N]
FDR	Final Drive Ratio
g	acceleration of gravity, [9.81 m/s ²]
\tilde{g}	average rate of vehicle fuel consumption over a driving schedule, [gal/mile]
g_0, g_i	combined rate of fuel consumption of a baseline vehicle, 0, and after the application of i independent technologies, respectively [gal/mile]
\tilde{g}_0, \tilde{g}_1	average rate of fuel consumption over a driving schedule for a baseline vehicle, 0, and after improvement, 1, [gal/mile]
g_c	combined rate of fuel consumption, [gal/mile]
$(g_c)_i$	combined rate of fuel consumption for the vehicle line i in the fleet, [gal/mile]
g_{fleet}	combined rate of fuel consumption for a fleet of vehicles using sales-weighted averaging, [gal/mile]
G_f	total volume of fuel consumed over a driving schedule, [gallons]
GR	Gear Ratio
\tilde{g}_U, \tilde{g}_H	average rate of fuel consumption for a vehicle driving the Urban and Highway schedules, respectively, [gal/mile]
H_f	heating value of the fuel per unit mass, [MJ/kg]
I_w	polar moment of inertia of a wheel, [kg m ²]
ISG	Integrated Starter-Generator
IVC	Intake Valve Control

$LIVC$	Late Intake Valve Closing	P_b	brake output power of the engine, [kW]
\tilde{m}	average vehicle fuel economy over a driving schedule, [miles/gal]	P_b^*	maximum (rated) brake output power of the engine, [kW]
M	test mass of a vehicle, [kg]	\bar{P}_b	time-averaged engine brake power for powered driving [kW]
m_C	combined fuel economy, [miles/gal]	$P_{b,ACC}$	brake power required by vehicle accessories, [kW]
m'_C	incorrect combined fuel economy when 55/45 weighting is applied to fuel economy rather than fuel consumption, [miles/gal]	$P_{b,max}$	maximum brake output power of the engine in a driving schedule, [kW]
\tilde{m}_0, \tilde{m}_1	average fuel economy over a driving schedule for a baseline vehicle, 0, and after improvement 1, [miles/gal]	P_f	power consumed by engine friction, [kW]
m_f	mass of fuel consumed in a driving schedule, [kg]	P_i	indicated power of an engine, [kW]
$(m_f)^+$	mass of fuel consumed in a driving schedule during powered driving, [kg]	P_p	pumping (gas exchange) power required in an engine, [kW]
$(m_f)_{brkg}$	mass of fuel consumed in a driving schedule during braking, [kg]	P_{TR}	instantaneous required tractive power, [kW]
$(m_f)_{idle}$	mass of fuel consumed in a driving schedule during idling, [kg]	$P_{TR,max}$	maximum tractive power required in a driving schedule, [kW]
\dot{m}_a	instantaneous air flow rate, [kg/s]	\bar{P}_{TR}	time-averaged tractive power during powered driving, [kW]
\dot{m}_f	instantaneous fuel flow rate, [kg/s]	\dot{Q}_{added}	rate at which energy is added to the thermodynamic cycle, [kJ/s]
$(\dot{m}_f)_{brkg}$	instantaneous fuel mass-flow rate during braking, [kg/s]	$\dot{Q}_{released}$	rate at which chemical energy is released by combustion, [kJ/s]
$(\overline{\dot{m}_f})_{brkg}$	time-averaged fuel mass flow rate during braking, [kg/s]	R	tire rolling-resistance force, [N]
$(\dot{m}_f)_{idle}$	fuel mass flow rate during idling, [kg/s]	r_0	tire rolling-resistance coefficient
m_{fleet}	combined fuel economy for a fleet of vehicles using sales-weighted averaging, [miles/gal]	r_w	effective rolling radius of wheel, [m]
\tilde{m}_U, \tilde{m}_H	average fuel economy of a vehicle driving the Urban and Highway schedules, respectively, [miles/gal]	r_c	engine compression ratio
n	number of vehicle lines in a manufacturer's fleet	S	total distance traveled during a driving schedule, [Urban = 12.00 km, Highway = 16.50 km]
N	engine speed, [rpm]		also, engine intake stroke, [mm]
N^*	engine speed at maximum power, [rpm]	S_e	engine effective intake stroke, [mm]
N_{idle}	engine idle speed, [rpm]	SR	torque-converter speed ratio
NRC	National Research Council	t	time, [s]
N_p	torque-converter pump speed, [rpm]	t_{idle}	duration of idle for an entire driving schedule, [Urban = 241 s, Highway = 4 s]
N_t	torque-converter turbine speed, [rpm]	t_{brkg}	duration of braking for an entire driving schedule, [s]
N_w	wheel speed, [rpm]	t_{0-60}	time required to go from a standing start to 60 mph at wide-open-throttle, [s]
P_a	power of auxiliaries essential to independent engine operation, [kW]	T	duration of a driving schedule, [Urban=1369 s, Highway=765 s]
		T_b	engine brake torque, [N m]

T_b^*	engine brake torque at maximum power, [N m]	$\eta_{b,max}$	maximum brake thermal efficiency
TDC	Top-Dead-Center piston position	η_{comb}	combustion efficiency
V	vehicle velocity, [m/s]	η_{dr}	instantaneous drivetrain efficiency
x_i	fraction of total sales for fleet i	$\tilde{\eta}_{dr}$	energy-transfer-weighted average drivetrain efficiency over the powered driving segment of a driving schedule, Eq. (16)
$(\% \tilde{g})$	decrease in average fuel consumption after a vehicle configuration change, [%]	η_i	indicated thermal efficiency
$(\% \tilde{m})$	increase in average fuel economy after a vehicle configuration change, [%]	η_{insul}	insulation efficiency
$(\%)_i$	average incremental reduction in fuel consumption of a baseline vehicle provided by technology i , [%]	η_{therm}	thermodynamic efficiency
α, β, γ	dimensional constants for tractive energy, Eq. (26) and Table 1	ξ_{rg}	overall regeneration effectiveness
α', β', γ'	dimensional constants for braking energy, Eq. (27) and Table 1	n / v	strokes per engine cycle divided by strokes per engine revolution, [rev/cycle]
γ	ratio of specific heat at constant pressure to that at constant volume	ρ	density of air, [1.20 kg/m ³]
η_b	instantaneous brake thermal efficiency of an engine	ρ_f	density of fuel, [0.735 kg/liter for gasoline]
$\tilde{\eta}_b$	fuel-consumption-weighted average brake thermal efficiency over the powered segment of a driving schedule, Eq. (15)	ρ_i	air density in the cylinder at the end of intake, [kg/m ³]
		τ	duration of powered driving for an entire driving schedule, [s]